ECF ECF5106/4106 CORPORATE FINANCE V

LECTURE 2

"On the theory of the optimal investment decision" - J Hirschleifer, JPE, 1958.

1. The article attempts to solve - through the use of isoquant analysis - the problem of optimal investment decisions - uses principles laid down by Irving Fisher. - analyses NPV and IRR rules.

2. The paper shows how Fisher's principles must be adapted

   - when borrowing and lending rates diverge.
   - when capital can be secured at an increasing marginal borrowing rate.
   - when capital is rationed.

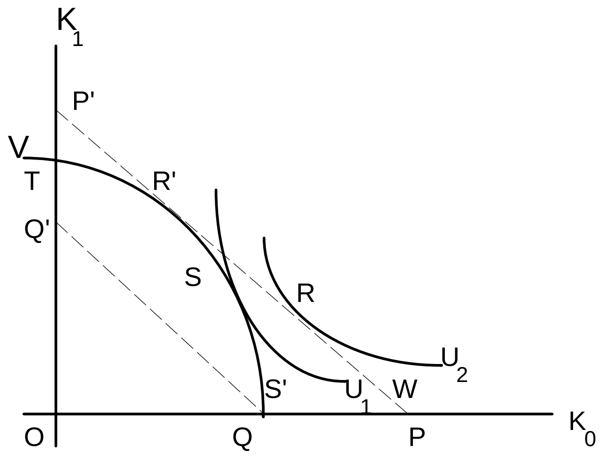
3. Section 3 provides the solution for multi-period investments - analyses the merits of the NPV and IRR rules.

- Limiting assumptions: the paper deals with a highly simplified situation in which the costs and returns of alternative individual investments are known with certainty - the problem being to select the scale and mix of investments to be undertaken.

- To begin with, the analysis will be limited to investment decisions relating to two time periods only.

Two period analysis: borrowing rate equals lending rate - Consider the case in which an individual can borrow or lend any given amount at one unique rate of interest. Fisher develops these conditions in the context of a perfect capital market. In the diagram, he measures current consumption or income and he measures period 1 income or consumption.

Fisher's solution



The individual's decision is to choose from the opportunities available to him - the optimum time pattern of consumption.

The starting point could be a point on either axis T or P or a point in the positive quadrant, i.e., W or S_i.

The individual has a preference function relating income in periods 0 and 1. Represented by indifference curves U_1 and U_2.

Finally, there are investment opportunities open to the individual. Fisher distinguishes between investment opportunities and market opportunities.

Thus we may invest by building a house or by lending on the money market.

In the diagram, an investor with a starting point at Q faces a market opportunity illustrated by Q.Q`. QQ` can be called a market line. The line PP` is the market line available to an individual where starting point is P on the $K_o$ axis, the line PP` is also the market line to someone whose starting point is W in the positive quadrant.

Finally, the curve QSTV shows the range of productive opportunities available to an individual with starting point Q - Fisher called this the productive opportunity curve. The concavity to the origin reveals a kind of diminishing return to investment.

Productive investments may be considered to be ranked by the expression $(\Delta k_1)/(-\Delta k_0)-1$, which might be called the productive rate of return.

QSTV could be viewed as a sequence of projects with the one yielding the highest productive return at the lower right.

The investor's objective is to climb on to as high an indifference curve as possible. Moving along the productive opportunity line QSTV, the highest indifference curve attainable is U_1 at S.

But the investor can do better if he moves up to point R` which is on the market line PP`. He can now move in reverse direction (borrowing) along PP` - and achieve point R on indifference curve U_2 - the best available.

The solution is in two steps - make the best investment from the productive point of view at R`, and then move along the market line to a point better satisfying his time preferences at R.

i.e., make the best investment from the productive point of view and then finance it in the loan market.

i.e., build a house - borrow against it on a mortgage so as to replenish current consumption.

Optimal decision rules

Present value rule - individual or firm should adopt all projects whose present value is positive at the market rate of interest.

Under the assumed conditions present value is K_0+(K1)/(1+i)

The market lines are lines of constant present value. The equation for these lines is K_0+(K1)/(1+i) = C, C being a parameter.

The present value rule tells us to invest until the highest such line is attained, i.e., point R` - the rule tells us nothing about the optimum financing necessary to achieve the final optimum at R. However, this does lead to an important **separation theorem**.

Internal Rate of Return Rule (IRR)

The firm or individual should adopt any project whose internal rate of return is greater than the market rate of interest, i.e., writing ρ for the IRR.

$$O = Dk_0 + \frac{Dk_1}{1+r} + \frac{Dk_2}{(1+r)^2} + \dots\dots\dots + \frac{Dkn}{(1+r)^n}$$

In the two period case ρ is identical with the productive rate of return.

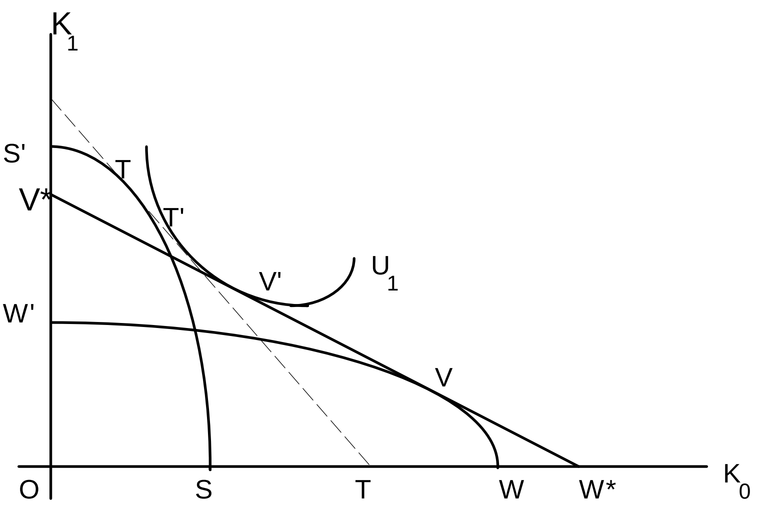
$(\Delta k_1)/(-\Delta k_0)-1$

i.e., the slope of the productive investment opportunity curve minus 1.

In the two-period case, the present value rule and the IRR give the same answers.

However, without a market opportunity line point, S would have been the optimum. When borrowing and lending rates differ, assume the borrowing rate is higher than the lending rate.

Fig. 2 - Extension of Fisher's solution for differing borrowing and lending rates.



There are now two sets of market lines on the graph - the steeper (dashed) lines represent borrowing opportunities - the flatter (solid) lines represent lending opportunities. The heavy curves show two sets of productive opportunities - both of which lead to solutions on U_1.

Starting with amount OW of K_0, an investor with a productive opportunity WVW1 would move to V and then he would lend to get to his time preference optimum.

The curve STS` represents a more productive opportunity. The investor would move to T and then borrow to move to T` his optimum.

The above diagram is divided into three zones: I, II and III. Tangency solutions at the market borrowing rate like that at T are carried back by borrowing back to a tangency with a utility isoquant at point T`. A similar solutions line along curve OB in this case - this connects all points on utility curves with slopes equal to the market borrowing line. Similarly, Zone 3 is the zone where solutions involve tangency with a market lending line like point V. OL connects all such points.

Finally, Zone II solutions occur when a productive opportunity locus like QRQ` is steeper than the lending rate but flatter than the borrowing rate. In Zone II, use of neither the borrowing nor the lending rate will lead to the correct answer.

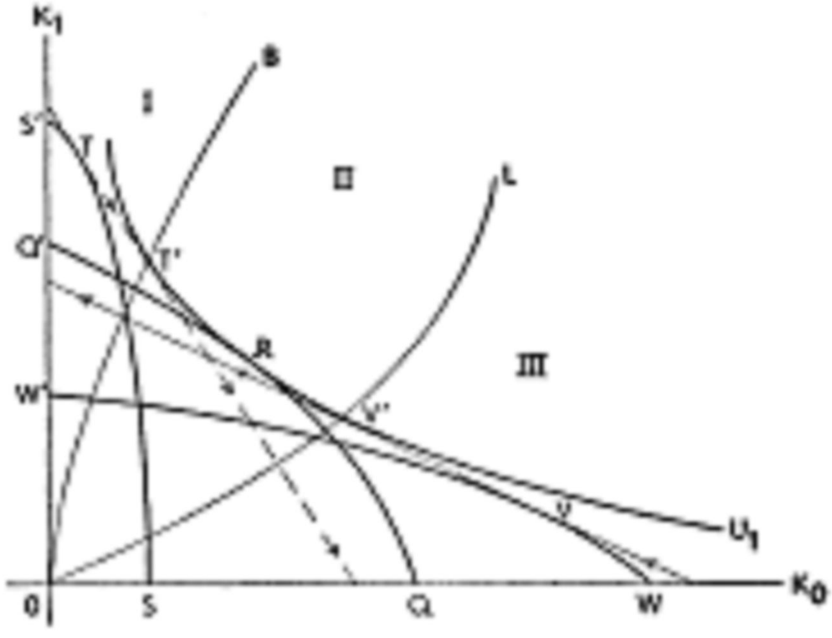


FIG. 3.—Three solution zones for differing borrowing and lending rates.

Increased Marginal Cost of Borrowing

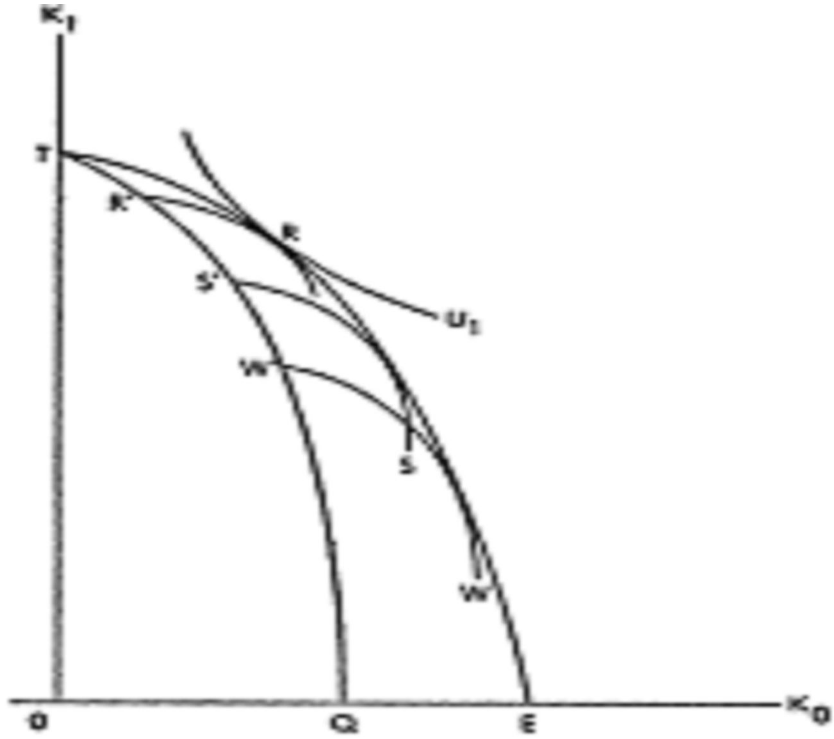


FIG. 4.—Increasing marginal cost of borrowing

The above diagram shows a productive opportunity locus QR`T and an indifference curve $U_1$. Assume marginal borrowing costs rise at the same rate, whether the investor begins to borrow at R` or W`.

The curve TE represents the total opportunity set as the envelope of these market courses, that is, TE connects all points on the market curves representing the maximum K_0 available for any given K_1. The optimum is found where TE is tangent to the highest attainable indifference curve, here the curve U_1 at R - this analysis applies in Zone 1 of the previous diagram. The marginal borrowing cost should be used as the discount rate in Zone 1 solutions, but we don't know what it is in advance - independently of the time preference function.

Hirschleifer agrees that F and V Lutz, *The Theory of Investment of the Firm*(1951) set off on the wrong foot - they begin by seeking intuitively for an optimum investment decision criterion and settle on "the maximization of the rate of return on the investor's owned capital". The

Fisherian approach - favoured by Hirschleifer - integrates the investment decision with the general theory of choice - the goal being to maximise utility subject to certain opportunities and constraints.

Hirschleifer is generally dismissive of the idea of capital rationing - we will return to this point. See Weingartner (1977). "Capital would be fixed to the firm only under rather peculiar conditions, specifically, if there is a discontinuity in the capital funds market such that the marginal borrowing rate suddenly becomes infinite at the firm's level of borrowings.

Non-independent investment opportunities

Suppose now there are two mutually exclusive sets of investment opportunities - i.e., building a factory in the east or the west but not both. The two sets of opportunities look like QV`V and QT`T in the diagram.

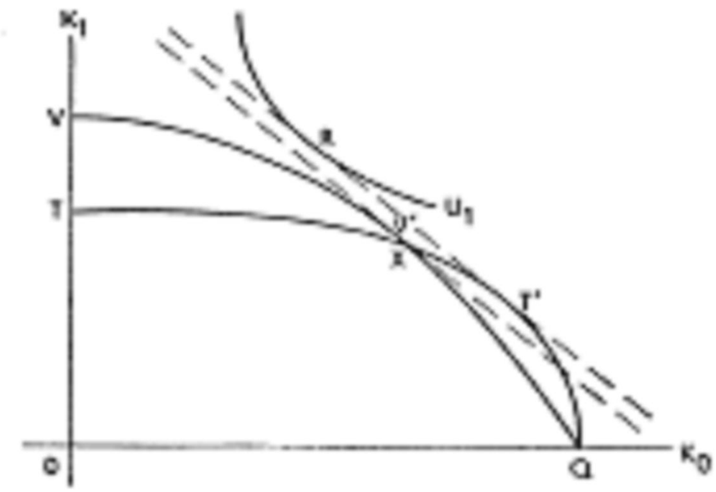


Fig. 5.—Non-independent investment opportunities—two alternative productive investment loci.

In the case of the perfect capital market with a unique borrowing-lending rate - go for the highest attainable present value T`, and then lend up to R. If the borrowing and lending rates differ, the problem becomes more complex and the rule may fall down.

Non-independent investment opportunities - poorer projects prerequisite to better ones

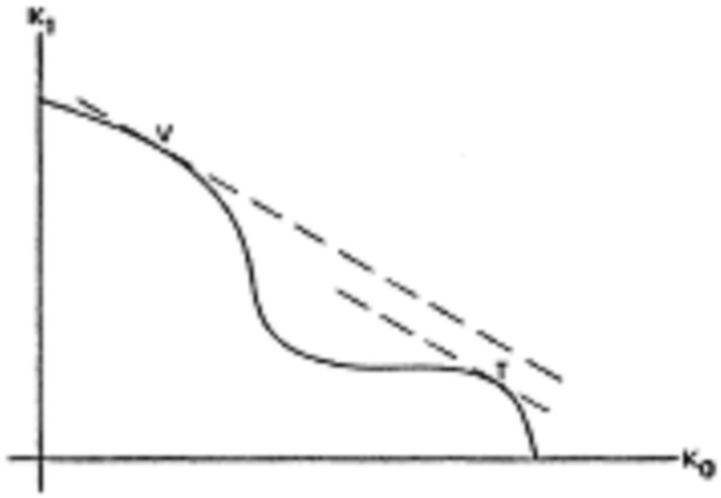


FIG. 6.—Non-independent investment opportunities—poorer projects prerequisite to better ones.

Possibility of local options like V and T - go for highest NPV.

Hirschleifer suggests - p.339

1. The IRR rule fails where there are multiple tangencies.

2. The NPV works wherever IRR does and also works in the case of multiple tangencies.

3. Both rules only work in a formal sense when a marginal opportunity rate is required.

4. The NPV can fail when a perfect capital market does not exist.

5. Even when the rules are correct in more than the formal sense, the answer given is a productive solution - only part of the way towards attainment of optimal utility.

Multiperiod Analysis

Equations of present value

$$K_0 + \frac{K_1}{(1+i_1)} + \frac{K_2}{(1+i)^2} + ...\frac{Kn}{(1+i)^n} = C$$

Failure of the IRR

The idea of a rate of growth involves a ratio and cannot be uniquely defined unless one can uniquely value initial and terminal positions. The investment option is characterised by cash flow -1.0.0.8 clearly involves a growth rate of 100% (compounding annually).

Consider the sequence -1.2.1 - no information is provided about the rate at which the intermediate receipt of 2 can be re-invested. If we adopt the IRR approach for the investment option -1.2.1 ρ is equal to √2 or 141.4% if the borrowing rate is less than √2 the investment is desirable.

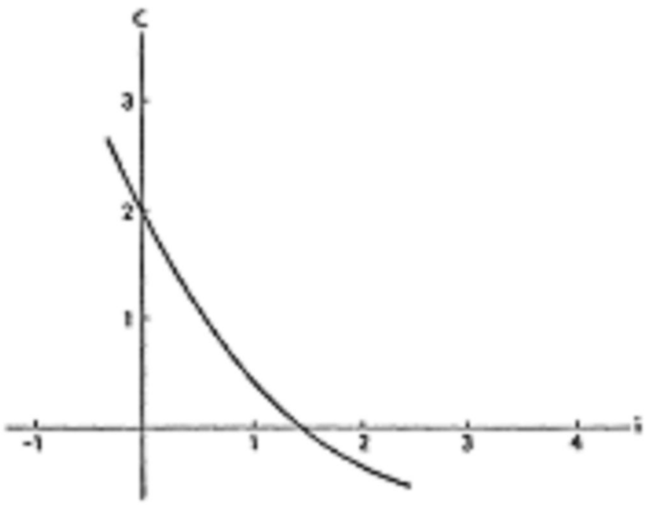


FIG. 7.—Sketch of present value of the option −1, 2, 1.

IRR does not necessarily lead to the correct decision rule. The decision cannot be made without the knowledge of the appropriate external discount rate. I is preferable for low rates of interest and II preferable for higher rates.

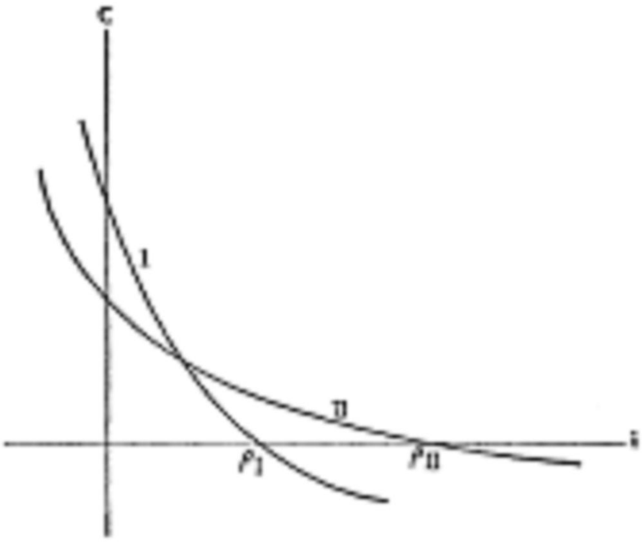


FIG. 8.—Two alternative options

IRR solutions at 100 and 200 percent may be even more positive roots - see discussion.

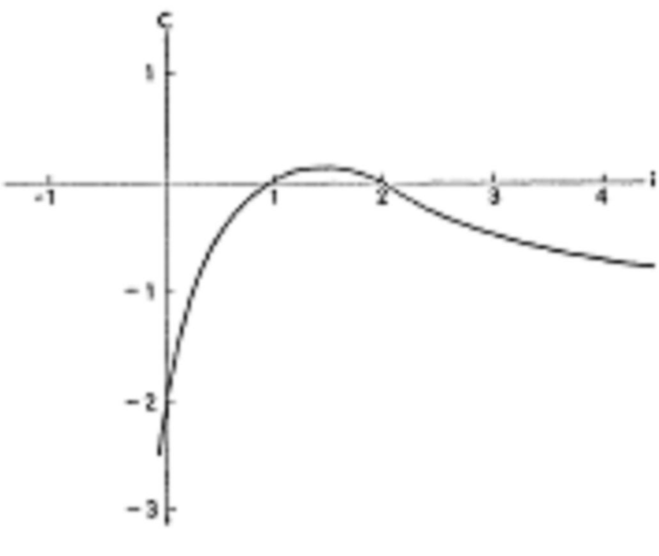


FIG. 9.—Sketch of present value of the investment option −1, 5, −6.

Hirschleifer concludes

The main positive conclusion of the paper is that the present value rule for investment decisions is current in a wide variety of cases (though not universally) and in a limited sense. The rule tells us to attain the highest possible level of present value. This is an intermediate productive solution which must then be modified by borrowing or lending to find the overall optimum.

Current extensions

In corporate finance, **real options analysis** or **ROA** applies put option and call option valuation techniques to capital budgeting decisions.

A **real option** is the right, but not the obligation, to undertake some business decision, typically the option to make a capital investment. For example, the opportunity to invest in the expansion of a firm's factory is a real option. In contrast to financial options, a real option is not often tradeable—e.g. the factory owner cannot sell the right to extend his factory to another party, only he can make this decision; however, some real options can be sold, e.g., ownership of a vacant lot of land is a real option to develop that land in the future. Some real options are proprietory (owned or exercisable by a single individual or a company); others are shared (can be exercised by many parties). Therefore, a project may have a portfolio of embedded real options; some of them can be mutually exclusive.

The terminology "real option" is relatively new, whereas business operators have been making capital investment decisions for centuries. However, the description of such opportunities as real options has occurred at the same time as thinking about such decisions in new, more analytically-based, ways. As such, the terminology "real option" is closely tied to these new methods. The term "real option" was coined by Professor Stewart Myers at the MIT Sloan School of Management; this happened most likely around 1977.

The concept of real options was popularized by Michael J. Mauboussin, the chief U.S. investment strategist for Credit Suisse First Boston and an adjunct professor of finance at the Columbia School of Business. Mauboussin uses real options in part to explain the gap between how the stock market prices some businesses and the "intrinsic value" for those businesses as calculated by traditional financial analysis, specifically discounted cash flows.

Additionally, with real option analysis, uncertainty inherent in investment projects is usually accounted for by risk-adjusting probabilities (a technique known as the equivalent martingale approach). Cash flows can then be discounted at the risk-free rate. With regular DCF analysis, on the other hand, this uncertainty is accounted for by adjusting the discount rate, using e.g. the cost of capital) or the cash flows (using certainty equivalents). These methods normally do not properly account for changes in risk over a project's lifecycle and fail to appropriately adapt the risk adjustment. More importantly, the real options approach forces decision makers to be more explicit about the assumptions underlying their projections.

Generally, the most widely used methods are: Closed form solutions, partial differential equations, and the binomial lattices. In business strategy, real options have been advanced by the construction of option space, where volatility is compared with value-to-cost, NPVq. Latest advances in real option valuation are models that incorporate fuzzy logic and option valuation in fuzzy real option valuation models.

Real options are a field of academic research, and at the present one of the leading names in academic real options is Professor Lenos Trigeorgis (U. of Cyprus).

- Real Options Tutorial, Prof. Marco Dias, PUC-Rio

Investment:

"Economics defines investment as the act of incurring an immediate cost in the expectation of future rewards" (Dixit & Pindyck, 1994, p.3)

Three important investment decision characteristics are relevant in Real Options Approach:

- Irreversibility
- Uncertainty
- Managerial Freedom Degree and Timing

Most investments are like a call option on a common stock, that gives the holder the right to make an investment and receive a project (see the call option analogy). The project value fluctuates stochastically and most investments options (or investment opportunities) are not a "now or never" opportunity. There is a value of waiting to invest. You will exercise the option (not the obligation) of investment only if the project is sufficiently "deep in the money". In others words, if the project's output price and/or the *dividend yield* of the project are sufficiently high.
Some financial options solutions may be useful in the real investments context (with some relevant adaptations and parameters interpretations) using analogies like the dividend yield analogy.

Investment's Return:

The return of an investment, like the return of a stock, comprises the addition of two parcels: the capital gain and the dividends. Over the time, the **expected rate of return** from a real investment is equal the sum of the **expected growth rate** (capital gain rate) plus the **dividend yield** (net cash flow distribution rate or the convenience yield of the underlying commodity, depending of context), see the next equation:

$$\mu = \alpha + \delta$$

The expected rate of return corresponds to the **"risk-adjusted" discounting rate**, from a financial market models like the CAPM (Capital Asset Pricing Model). In presence of managerial flexibility the discounting rate changes over time, and the DCF approach becomes quite complex and inadequate.

Irreversibility:

Investments in productive capacity are, in general, irreversible: you cannot recover all the money, if the business doesn't become prosperous. The cost of investment is partially or completely sunk.

Petroleum examples:

- Completely Sunk:
    - the drilling of one well is completely sunk (the cost of recover material like casings is more or equal the material value itself). If the well productivity is

worse than you expected or if the oil's price falls, nobody get back the money spent; and

- offshore pipeline (rigid material): the material recovering cost is more or equal the material value itself.

- Partially Sunk:

  - the subsea well's completion (production equipment installation into the subsea well) is partially irreversible because the equipment named Wet Christmas Tree (WCT) owns a high value and the net money recovered may reach 20 to 30% from the total *well's completion* investment; and

  - offshore flexible pipeline is partially irreversible. In opposition to the rigid pipeline, the flexible material is more expensive but the service cost is cheaper, so there is a valuable switching option in the flexible pipelines case.

But even in the partially sunk examples, there is a alternative value for the second-hand material only if the problem is technical (a well with low productivity or an exhausted short-lived reserve, for example). If the problem is economic (a depressing oil's price affecting the industry as a whole), the recovered material value is reduced to a little more than the scrap value (the "little" is because the option value of the material to be valuable again in next future).

There are others equipment that are valuable (much more than scrap value) even in large industry specific crisis. Examples are electric generators, computers, trucks, and so on, but the "market for lemons" effect lowers the real value of the equipments. If the industry crisis is not specific (is a consequence from a world's recession), even these less specific equipments values are affected (depressed) by the lower demand.

Uncertainty:

Some few concepts:

Certainty : "refers to situations when the investor knows with probability 1 what the return on his investment is going to be in the future" (Levy & Sarnat, 1984, p.77). So ***uncertainty*** is when a collection of values (associated with respective uncertain "states of nature") can happen, with strictly positive probabilities for, at least, two different possible values.
Investors are assumed to have ***rational expectations*** in that they agree on the mappings of the assets values (for all states-of-nature along the time). See for example Huang & Litzemberger (1988, p.188). In others words, they have rational expectations about the underlying stochastic processes of the assets. This is an equilibrium model under uncertainty. See also the source Lucas & Prescott (1971) and Muth (1961) This concept is more powerful than the *adaptive expectations* from past prices or the *static expectations* concept that underlies the traditional Marshallian's microeconomic theory. For this last point and for an example of the *rational expectations* at work, see (Dixit & Pindyck, 1994, pp.219-221). The problem is to maximize the firm's wealth, supposing as given the assets' stochastic processes, that all the investors with rational expectations agree when reaching the prices equilibrium.
Although some authors make a theoretic distinction between risk and uncertainty, in finance practice these terms are used with the same meaning. From a practical point of view, I prefer the term *uncertainty* because its neutral connotation is more appropriated for a scientific economic study (*risk* has frequently a negative connotation, or emphasizes the "bad side" of the uncertainty). The term "risk" is usual (and useful) in financial markets and financial operations (hedging, dividend and debt policy) in corporations, not in economic decisions.

GLOSSARY AND ADDITIONAL DEFINITIONS

- **Call Option:** gives the holder the right (not the obligation) to buy the *underlying asset* by a certain *expiration date* for a certain *exercise price*.
  In finance the *underlying asset* could be stocks, commodities contracts, futures contracts, foreign currencies, stock indices, and so on. In real investments context, the value of the implanted project (after the investment) is the underlying asset, and the net cash flows represents the dividends (see also the dividend yield analogy and the call option analogy).
  There are two kinds of calls: "American Call Options" and "European Call Options".

- **Theoretic Distinction Between Risk and Uncertainty:** as Simonsen (1994, p.399) and Levy & Sarnat, 1984, p.104) point out, the distinction between risk and uncertainty is creditable to F. Knight (in *Risk, Uncertainty and Profit*, 1921, Chapter VII). Risk is when the random variable has a *known* probability distribution. Uncertainty is when the random variable has an *unknown* probability distribution. It is always possible to convert uncertainties into risks by introducing subjective probabilities. Levy & Sarnat (p.106) argues that "In the case of financial investment, probability beliefs are almost invariably subjective" and in the book they "use the terms *risk* and *uncertainty* interchangeably".

- **The "Bad News Principle":** In a world with uncertainty , a "good" project (with positive net present value) can be postponed if a firm can wait to invest, because of the "bad news principle": there is a positive probability of a downward price movement, so that the waiting avoids the possibly losses from the project investment. The threshold price (that warrants immediate investment) depends of the size of the downward movement (not of the upward movement size) and the probability (besides the investment cost, of course), as shown in the equation 16, p.41, in the Dixit & Pindyck book.
  This "bad news principle" was first pointed out by Bernanke, 1983, and some ideas can also be found in Cukierman, 1980.
  A nice and instructive argument is presented by Dixit, 1992 (see p.123), illustrating the "bad news principle": Why the Japanese firms (despite their larger fixed costs) are more aggressive investors than the American ones? Because they are protected from the downside risk through the government supports. So the value of waiting to invest is quite small, because the downside movement potential is small.
  For disinvestment (abandon) the argument turns around and becomes the **"good news principle"**: the option value of keeping the operation alive is governed mainly by the upside potential.

The Two Sides of the Uncertainty and the Asymmetry Effects:
Uncertainty means for example, that the future price of oil's barrel will be **up or down**, in relation to the forecasted price. So uncertainty has two sides: the "good" side and the "bad" one.

Rational managers are not passive: management can revise investment and operating decisions in response to market conditions, in order to maximize the firm's wealth. They act to take advantage in "good times" (market's upside) and mitigate losses in "bad times" (market's

downside). So, in presence of economic uncertainty, active management adds value to investment opportunity, which is not captured by the traditional use of discounted cash flows (DCF) method. See for example Trigeorgis & Mason (1987, p.15)
For example, the ability to wait permits that the manager "see" the evolution of the oil prices, before takes an irreversible decision to invest in a new oilfield or abandon an old oilfield: if the oil's price increases to a sufficiently high level (the good side of uncertainty), the manager makes the investment in the new oilfield (in better conditions or with less probability of losses) or reopen the old oilfield. However, if the oil's price decreases (the bad side of uncertainty), the manager doesn't invest in the new oilfield and can abandon the old oilfield (at a sufficiently low price).
If the uncertainty is technical, for example a R&D investment, again the uncertainty adds value to this opportunity: a step by step investment reveals informations. So, a rational manager will stop the project (or reduce the investment) if the information is unfavorable (bad side), and continue the investment (or even speed it up) if is a favorable one (good side).
So, an increase of uncertainty, increases the investment opportunity value (the opposite that tells the traditional DCF) in view of the asymmetric manager's action in response to uncertainty. This is **the asymmetry on the value** of the opportunity to invest in a project (or option to invest), the first asymmetric effect of the uncertainty.
However, increasing the value of the option to invest doesn't mean increasing the willingness to invest: an increase of *economic* uncertainty reduces the willingness to invest (or delays the investment decision), because the increment in the investment opportunity value is due to the waiting value.
What side of uncertainty is more important for decisions? The answer is: depends on the kind of decision to be taken (see, in the "Real Options by Slides" page, the picture "The Managerial Decisions"). If the decision is to **invest or wait**, the bad side is the relevant one, and the decision is governed by the "bad news principle" (or fear/caution principle).
If the decision is to **abandon or wait**, the good side is the relevant one, and the decision is governed by the "good news principle" (or the hope principle). See for example Dixit & Pindyck, 1994 , pp.18 and 40-41.
These different roles played by the two sides of the uncertainty is the **asymmetry on the decision rule**, the second asymmetric effect of the uncertainty.

Economic Uncertainty and **Technical Uncertainty**
There are two types of uncertainty which has different (opposite) effects on investment decision rule:
The ***economic uncertainty*** is correlated with the general movements of the economy (industry's prices/costs movements). So the oil price (or the reserve price, or the rig rental rates, etc.) is an example of variable with economic uncertainty. This uncertainty is *exogenous* to the decision process: the oil price (or its variance) doesn't change *because* you go ahead with the oilfield development or not (supposing a decision from a *price taker* firm, not an OPEC decision). The economic uncertainty incentives the waiting (for better conditions) to invest, leading to postpone investments. A project with a considerable *positive* Net Present Value (NPV) can be insufficient to immediate investment: it is necessary that the project to be "deep in the money".
The ***technical uncertainty*** is NOT correlated with the general movements of the economy/industry. This uncertainty is *endogenous* to the decision process. An example is a non-delimited new oilfield: the oil-in-place volume (or the gas, or the water-in-place, or the average permeability, etc.) is a variable with technical uncertainty. In this case, waiting doesn't change the variable value. Only with a step-by-step investment strategy (3D-seismic, drilling some wells, pilot production, etc.) is possible to reduce this kind of uncertainty. So investing (step-by-step) provides valuable information (reduce the variance of the uncertainty and revise

the expected value). This additional value is called a *shadow value*, because it is not a directly measurable cash flow (traditional DCF doesn't "see" this). The technical uncertainty, on the contrary of the economic uncertainty, incentives the starting of the investments (but its necessary to be staged investments). In presence of relevant technical uncertainty, a project with *negative* NPV, can be economically optimal to begin investing. For each new relevant information is necessary to revise the investment decision: go ahead (or even speed it up) if the true value is on the "good side" of uncertainty, or stop if the "bad side" is the true one.
For a modern technical uncertainty explanation, including a model with the two types of uncertainty combined into the same variable (cost), see Dixit & Pindyck, 1994 , pp.11, 47-48, Chapter 10/section 4, and p.397 footnote 2) and also Pindyck, R.S. (1993)
One last question: What the difference between a investor holding a common stocks' portfolio, and a manager with a projects' portfolio? In the first case the investors can't take advantage of the technical uncertainty (investors can only change the portfolio composition, and eliminate this kind of uncertainty by diversification, cannot maximize the value of each stock), but in the second case the managers can take advantage of this, with optimal management of these projects. Managers don't want eliminates technical uncertainty by diversification, they want maximize the firm value by taking advantage of this one.
So technical uncertainty is important only for managers.

Uncertainty and Hedge: It's very known the Modigliani & Miller (MM) proposition II (see MM's propositions). There is another example of application for the MM's theorem, as indicated by Dixit & Pindyck (1994, p.11), for the investment opportunity (or investment *option*): "the option value is not affected if the firm is able to hedge the risk by trading in forward or futures market. In efficient markets such risk is fairly priced, so **any decrease in risk is offset by the decrease in return**" and the financial operation "has no effect on firm's real decisions".
For non-perfect markets, in general is possible to say that financial operations (like hedge) has a secondary (second-order) effect in real investment of corporations. So first the firm takes an ***economic decision*** about the production project investment (most cases a preference-free decision). After this, the firm takes the *financial and/or hedging decision* (most cases a preference-dependent decision).
Here has NOT been saying the financial decision is unimportant. Only argues this is a separated/independent decision that takes place after the investment (economic) decision. Hedging and Capital Structure aren't scope of this site.

Call Option Analogy:

An irreversible investment opportunity (F) is like a financial call option: the manager can (but is not obligated) spend the investment cost (D) to obtain a production asset (V), and frequently this investment opportunity remains only by a time interval (T-t).
The financial call option is like F, their exercise price is like D, the stock (received with the exercise of the option) is like V, and the call's expiration time is like (T-t). The financial analogy is more adequate with financial assets paying a continuous dividends (or interest) because the analogy with cash flows (see below the dividend yield analogy).
The expiration time in real investment can be the time from a patent's right, the time from a lease in a offshore tract, or an estimate time considering the threat of preemption or intense industry rivalry (for this last point, see Kester, 1984).
However, there are some differences and considerations when performing this analogy: Sick, 1995 (abstract) gives some lessons: "the early exercise decision is more important in real options analysis, greater flexibility in modelling project is needed" and his instructive practical lesson: **"The ability to be able to build a useful and understandable model, is more important to the analyst than precise estimates of option value."**

Dividend Yield Analogy:

The net cash flows from a production project is analogous to a dividend from a stock, or a periodic revenue from assets in general.
There is an analogy between the dividend yield from a project and the dividend yield characterized by the currency interest rate, in the currency option market, for example, because the dividends are close to continually distributed over the time. This analogy is more general: the option in a project paying a continuous net cash flow is analogous with the options written on commodities and commodity futures contracts. The underlying commodity varies from a natural resource like petroleum or silver to a financial issue such as foreign currency or a Treasury bond. Each case has its particulars adaptations, but the general understanding is the same
In most cases, is reasonable to consider the dividend yield from the project as a constant even when the cash flows falls over the time (like a depletive oil field), because the project value (the oilfield value) also decreases, so that the percentage variations can be considered more or less the same (or when using an average value is reasonable). With a constant dividend yield, a simplified solution could be used, like an analytic approximation. Some analytic approximation models are available in the literature on financial options that could be used in the real options context (see in the bibliographical page). Others approaches probably just find numerical solutions, such as interesting models that consider the dividend yield as a function of the underlying project value or even models considering the dividend yield as stochastic process itself, correlated with the project's value stochastic process.

BIBLIOGRAPHICAL REFERENCES

- **Bernanke, B.S. (1983):** *Irreversibility, Uncertainty, and Cyclical Investment* Quarterly Journal of Economics, 98, February 1983, pp.85-106

- **Brealey, R.A. & S.C. Myers (1991):** *Principles of Corporate Finance* McGraw-Hill, Inc., fourth ed., 1991

- **Cukierman, A. (1980):** *The Effects of Uncertainty on Investment under Risk Neutrality with Endogenous Information* Journal of Political Economy, vol.88, June 1980, pp.462-475

- **Dixit, A.K. (1992):** *Investment and Hysteresis* Journal of Economic Perspectives, vol.6, no 1, Winter 1992, pp.107-132

- **Dixit, A.K. & R.S. Pindyck (1994):** *Investment under Uncertainty* Princeton University Press, Princeton, N.J., 1994

- **Huang & Litzemberger (1988):** *Foundations for Financial Economics* Elsevier Science Publishers Co., Inc., 1988

- **Kester, W.C. (1984):** *Today's Options for Tomorrow's Growth* Harvard Business Review, no 62, March-April 1984, pp.153-160

- **Levy, H. & M. Sarnat (1984):** *Portfolio and Investment Selection: Theory and Practice* Prentice-Hall International, Englewood Cliffs, NJ, 1984

- **Lucas, R.E. Jr. & E.C. Prescott (1971):** *Investment under Uncertainty* Econometrica, vol.39, no 5, September 1971, pgs.659-681

- **Modigliani, F & M.H. Miller (1958):** *The Cost of Capital, Corporation Finance, and the Theory of Investment* American Economic Review, vol.48, June 1958, pp.261-297

- **Muth (1961):** *Rational Expectations and the Theory of Price Movements* Econometrica, vol. 29, 1961, pp. 315-335

- **Pindyck, R.S. (1993):** *Investments of Uncertain Cost* Journal of Financial Economics, vol. 34, August 1993, pp.53-76

- **Sick, G. (1995):** *Real Options* Paper presented at INFORMS International Singapore, June 25-28, 1995, 70 pp.

- **Trigeorgis, L. & S.P. Mason (1987):** *Valuing Managerial Flexibility* Midland Corporate Finance Journal, Spring 1987, pp.14-21