FINANCE CORPORATE FINANCE V ECF5101

LECTURE 4

In the maximisation of shareholder wealth, it has been suggested that the company's decisions can be divided into four groups:

a) How should the company finance its investment?
b) How should the company distribute its revenue?
c) How much should the firm invest?
d) Which projects should the firm undertake (or what techniques of production should it use)?

For the last couple of weeks, we have been concerned with (c) and (d). We now move on to (a). The first two questions above concern financial decisions. The latter two investment decisions. In perfect capital markets, the two sets of decisions can be distinguished and treated independently.

Assume a hypothetical firm financed purely by long-term debt and equity. V is the company's value, S the market value of its equity, and B the market value of its debt.

$$V = S + B \qquad (1)$$

Assume a perfect market with no frictions or transaction costs - also assume that

a) The expectations of investors in the market are the same as for current earnings - they are not expected to grow.
b) All existing and future investments are regarded as having the same risk.

X: annual expected net operating earnings

F: annual debt interest

Y: annual net earnings on equity

The equity capitalisation rate is given by

$$Ke = \frac{Y}{S} = \frac{\text{earnings available to equity}}{\text{market value of equity}} \quad (2)$$

The debt capitalisation rate is given by

$$Ki = \frac{F}{B} = \frac{\text{debt interest}}{\text{market value of debt}} \quad (3)$$

The cost of capital is given by

$$Ko = \frac{X}{V} = \frac{\text{net operating earnings}}{\text{total market value of company}} \quad (4)$$

The cost of capital Ko is a weighted average cost of capital

$$Ko = W_1 Ke + W_2 Ki \quad (5)$$

Where W_1 is the proportion of equity in the capital structure and W_2 is the proportion of debt.

$$1 = W_1 + W_2$$

It follows that

$$Ko = Ke\frac{S}{(S+B)} + \frac{KiB}{(S+B)} \quad (6)$$

What happens to Ko, Ke, and Ki when the level of leverage as measured by $\frac{B}{S+B}$ is changed?

The Net Income Approach

In this approach, it is assumed that both the interest rate on debt and the rate at which shareholders capitalise net earnings to shareholders are constant, regardless of the level of gearing or leverage.

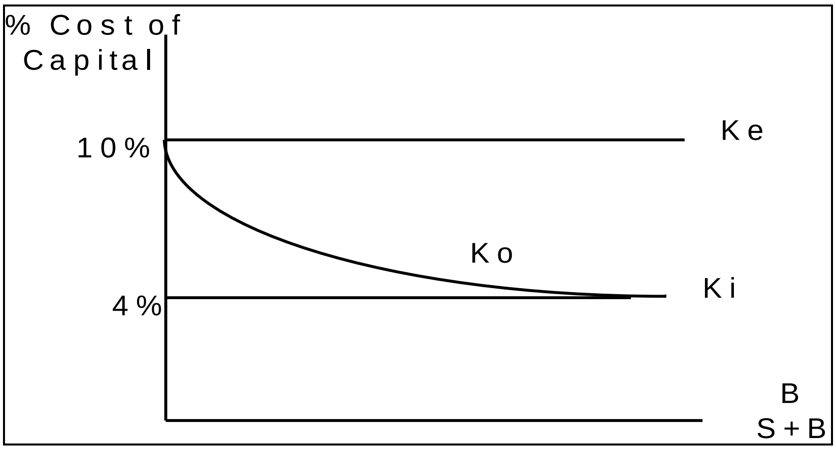
Assume there exists a no growth firm with annual net operating income (NOI) equal to $1000. Its equity capitalisation rate (Ke) is 10% and it is financed entirely by equity.

X	=	NOI	=	$1000	=	Net Operating Income
		F	=	0	=	Debt Interest
Y	=	NI	=	$1000	=	Net Income
Ke	=	10%	=		=	Equity Capitalisation Rate
		V	=	$10,000	=	Market Value of Company

Now assume the company levers up and 'replaces' $3,000 of equity with debt with an interest rate of 4%.

X	=	NOI	=	$1000	=	Net Operating Income
F	=	4% x 3000	=	$120	=	Debt Interest
Y	=	NI	=	$880	=	Net Income
Ke	=	10%	=		=	Equity Capitalisation Rate
		S	=	$8,000	=	Market Value of Equity
		B	=	$3,000	=	Market Value of Debt
V	=	S + B	=	$11,800	=	Market Value of Company

The assumptions imply that the company's value increases with increases in leverage



The relationship between leverage and cost of capital: the net income approach.

$$\text{Ko} = \text{Cost of Capital} = \frac{\text{X}}{\text{V}} = \frac{\text{net operating earnings}}{\text{market value of company}}$$

$$\text{Ko} = \frac{\$1{,}000}{\$11{,}800}$$

$$\therefore \text{Ko} = 8.47\%$$

The implication is that reductions in leverage reduce the cost of capital.

The Net Operating Income Approach

The basis of this approach is that the average cost of capital is constant regardless of the degree of leverage. It is assumed that if Ki<Ke, Ke will increase as leverage is increased.

$$\text{Ke} = \text{Ko} + (\text{Ko} - \text{Ki})\frac{\text{B}}{\text{S}} \qquad (7)$$

If we take the previous example

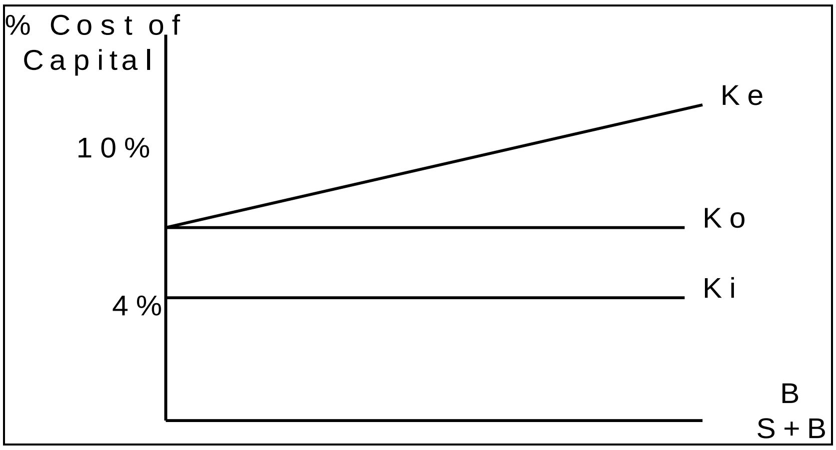
X	=	NOI	=	\$1000	=	Net Operating Income
Ko = Ke	=	10%	=		=	Equity Capitalisation Rate
V	=	S	=	\$10,000	=	Market Value of the Company

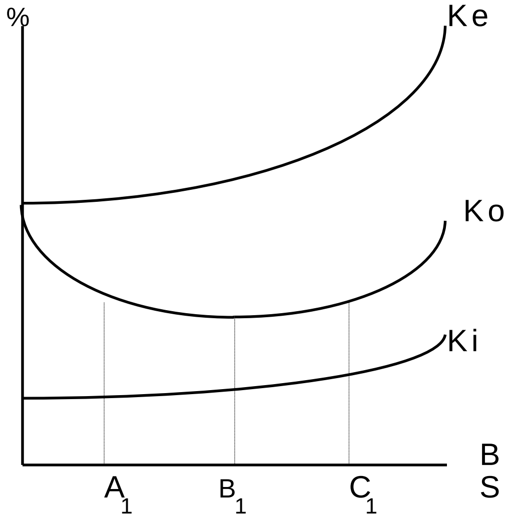
Again assume the firm replaces \$3,000 of equity with debt.

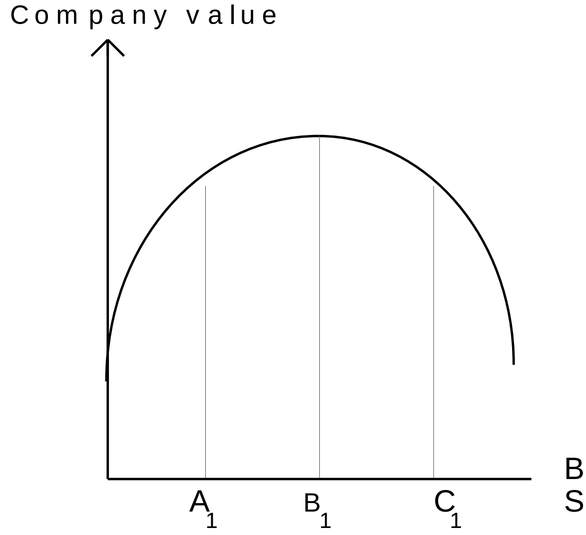
X =	NOI	=	$1000	=	Net Operating Income
Ko =	10%	=		=	Average Cost of Capital
	V	=	$10,000	=	Company Value
	B	=	$3,000	=	Value of Debentures
	S =V-B	=	$7,000	=	Value of Equity

If the interest on the debentures is $120, this leaves $880 for the equity holders. This means the equity capitalisation rate has risen to

$$Ke = \frac{Y}{S} = \frac{NI}{S} = \frac{\$880}{\$7,000} = 12.57\%$$







M&M 1958

Assume a situation of uncertainty in which investors agree on the means of the probability distribution of earnings, that firms can be divided into equivalent return classes of similar risk.

In these classes, if Pj denotes the price and Xj is the expected return pv the firm of class k, and the cost of capital is $\frac{1}{\rho k}$

we have $pj = \frac{1}{\rho k}\bar{x}_j$ (1)

or equivalently $\frac{\bar{x}_j}{p_j} = \rho_k$ (2)

Proposition 1

Consider any company j, let $\bar{x}_j$ stand for expected profile before the deduction of interest, D_j is the market value of the debt, S_j the market value of the shares.

$\therefore V_j \equiv S_j + D_j$. In equilibrium, proposition 1 suggests that

$$V_j = \left(S_j + D_j\right) = \frac{\bar{x}_j}{\rho k} \quad (3)$$

for any firm j in class k.

That is, the market value of any firm is independent of its capital structure and is given by capitalising its expected return at rate ρk appropriate to its class.

This proposition can be stated in an equivalent way in terms of the firm's 'average cost of capital' $\frac{\bar{x}_j}{v_j}$, which is the ratio of its expected return to the market value of all its securities. Our proposition then is:

$$\frac{\bar{x}_j}{\left(S_j + D_j\right)} = \frac{\bar{x}_j}{v_j} = \rho k \qquad (4)$$

for any firm j in class k.

That is, the average cost of capital to any firm is completely independent of its capital structure and is equal to the capitalisation rate of a pure equity stream of its class.

To establish proposition 1, we will show that as long as the relations (3) and (4) do not hold between any pairs of firms in a class, arbitrage will take place and restore the stated equalities.

Example

Suppose there are two companies, A and B, both in the same equivalent return class and both earning $1000 net operating income. The only difference between the two companies lies in their capital structure, as company A is financed purely by equity, whereas $3,000 of debit in its capital structure. It is assumed that the equity capitalisation Ke is 10% and the interest rate Ki is 4%. Assume the following disequilibrium situation,

			Company A	**Company B**
X	=	net operating income =	$1,000	$1,000
F	=	debt interest	0	$120
			______	______

Y =	net income =	\$1,000	\$880
Ke =	equity capitalisation rate	0.10	0.10
S =	equity value	\$10,000	\$8,880
B =	market value of debt	0	3,000
V =	S+B = Company value	\$10,000	\$11,800
Ko =	$\frac{X}{V}$ = cost of capital =	10%	8.47%
$\frac{B}{S}$ =	gearing ratio =	0	0.375

Company B Value>Company A Value - a disequilibrium situation.

Suppose a rational investor owns 10% of Company B worth \$880. He should sell these and substitute his own gearing in the same ratio, i.e., borrow \$300 at 4%. He now has funds worth \$1,180. He should reinvest these by spending \$1,100 on 11.8% of Company A - this will make him better.

Original income

A 10% holding in Company B yields an annual income of \$88.

Income after arbitrage

An 11.8% holding in Company A yields an annual income of	\$118
Less interest at 4% on borrowings of \$300	-\$12
His new income after arbitrage	\$106

Clearly he is better off after this arbitrage - this would push the value of B shares down until they are worth \$7,000. This is the equilibrium position. If he sold 10% of A worth \$700 and borrowed \$300 he would have total funds of \$1,000.

Original income

Annual income from a 10% holding of Company B shares is $88.

Income after arbitrage

A 10% holding in Company A yields an annual income of	$100
Less interest at 4% on borrowings of $300	-$12
His new income after 'arbitrage'	$88

Proposition II

From proposition I we can derive the following proposition about equity rates of return I on the stock of any company j belonging to the kth class.

$$i_j = \rho_k + (\rho_k - r) D_j / S_j \qquad (5)$$

The arbitrage mechanism ensures that proposition II holds.

M&M 1963

Taxes leverage and the probability distribution of after-tax returns

Let's denote X the (long run) average earnings before interest and taxes of a given firm in risk class K. X can be expressed as $\overline{X}Z$where $\overline{X}$ is the expected value of x and the random variable Z = $X/\overline{X}$, having the same value for all firms in class K, is a drawing from a distribution say fk(Z). Hence the random variable X^t measuring the after-tax return can be expressed as

$$X^t = (1-t)(X-R)+R = (1-t)X + tR = (1-t)\overline{X}Z + tR \qquad (1)$$

where t is the marginal corporate income tax rate I and R is the interest bill. Since $E(K^r) \equiv (i-t)\overline{X} + tR$, we can substitute this in (1) to obtain

$$X^t = (X^t - tR)Z + tR = \overline{X}^t\left(1 - \frac{tR}{X^t}\right)Z + tR \qquad (2)$$

This means that if the tax rate is non zero, the shape of the distribution of X^t will be influenced by the tax rate and the degree of leverage, for example if var(Z) = σ^2 we have

$$\mathrm{Var}\left(X^t\right) = \sigma^2\left(\overline{X}^t\right)^2\left(1 - t\frac{R}{\overline{X}^t}\right)^2$$

which means that for given X^t the variance is smaller the higher t and the degree of leverage.

The valuation of after-tax returns

From (1) the long-run average stream of after-tax returns appears as the sum of two components: (1) an uncertain stream $(1-t)\overline{X}Z$ and (2) a sure stream tR.

In the case of an unlevered company, this implies the value is

$$\rho^t = \frac{(1-t)\overline{X}}{Vu} \text{ or } Vu = \frac{(1-t)\overline{X}}{\rho^t}$$

Thus, we would expect the value of a levered firm of size $\overline{X}$ with a permanent level of debt D_L in its capital structure to be

$$V_L = \frac{(1-t)\overline{X}}{\rho^t} + \frac{tR}{r} = Vu + D_L \qquad (3)$$

Some implications of formula (3)

With leverage the extra after-tax earnings are capitalised at r the certainty rate.

Consider the before-tax earnings yield, the ratio of before-tax expected earnings to the value of the firm. Dividing both sides of j(3) by Val (1-t) and simplifying, M&M obtain

$$\frac{\overline{X}}{V} = \frac{\rho^t}{1-t}\left[1 - t\frac{D}{V}\right]$$

Consider the after-tax earnings yield.

We substitute $\overline{X}r - tR$ for $(1-t)\overline{X}$ in (3), obtaining

$$V = \frac{\overline{X}_t - tR}{\rho^t} + tD = \frac{\overline{X}_t}{\rho^t} + t\frac{\rho^t - r}{\rho^t}D$$

from which it follows that the after-tax earnings yield must be

$$\frac{\overline{X}^t}{V} = \rho^t - t(\rho^t - r)\frac{D}{V} \tag{4}$$

Finally, the after-tax yield on equity capital by subtracting D from both sides of (4) and breaking $\overline{X}^t$ into its two components, expected net profits after taxes Π^t and interest payments R=rD. They obtain, after simplifying

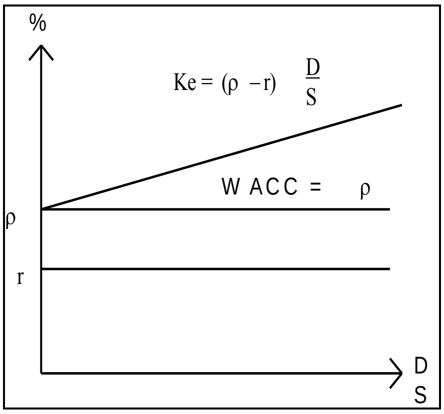
$$S = V - D = \frac{\Pi^t}{\rho^t} - (1-t)\frac{(\rho^t - r)}{\rho^t}D \tag{5}$$

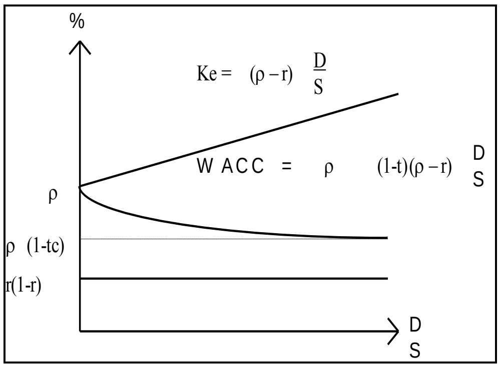
It follows from (5) that the after-tax yield on equity capital must be

$$\frac{\overline{\Pi}^t}{S} = \rho^t + (1-t)[\rho^t - r]\frac{D}{S} \tag{6}$$

Contrast this with the original proposition II which said Ke = $\rho_k + (\rho_k - r)\frac{D_j}{S_i}$

Cost of capital with no taxes　　　　Cost of capital with taxes





Taxes and the cost of capital

The minimum required return on an investment project can be defined before or net of taxes - they concentrate on the before-tax case, in effect, they are funding the minimum value of $d\bar{X}/dI$ for which dV = dI, where I denotes the level of new investment.

By differentiating (3), they obtain

$$\frac{dv}{dI} = \frac{1-t}{\rho^t}\frac{d\bar{X}}{dI} + t\frac{dD}{dI} \geq 1 \text{ if } \frac{d\bar{X}}{dI} \geq \frac{1-t^{\frac{dD}{dI}}}{1-t}\rho t \tag{7}$$

Hence the before-tax required rate of return cannot be defined without reference to financial policy.

For a new project entirely financed by equity, the required rate of return from (7) would be

$$\rho^s = \frac{\rho^t}{1-t}$$

but for one entirely financed by debt

$$\rho^D = \rho^t$$

M&M suggest a compromise in which a long-run target leverage level L* is assumed to exist and new investments are financed with this mix of debt and equity. This means $dD/dI = L^*$.

They define a WACC ρ* using this target mix of debt and equity:

$$\rho^* = \frac{(1-tL^*)}{1-t}\rho t = \rho^s - \frac{t}{1-t}\rho^D L^*$$

$$= \rho^s(1-L^*) + \rho^D L^*$$

Copeland and Western, Chapter 13, discuss how the above approach is consistent with the standard treatment of the weighted average cost of capital.