CORPORATE FINANCE ECF5101

LECTURE 3a

M&M Dividend Policy

Assumptions:

1. Perfect capital markets - no frictions, taxes, etc.
2. Rational behaviour
3. Perfect certainty about future investment plans.

Let dj(t) = dividend per share point by firming during period t

Pj(t) = the price (ex div) in t-1) of a share in firm j at the start of t.

We must have

$$\frac{dj(t)+Pj(t+1)-Pj(t)}{Pj(t)} = \rho(t) \qquad 1$$

or equivalently

$$Pj = \frac{1}{1+\rho(t)}[dj(t)+Pj(t+1)] \qquad 2$$

The effect of dividend policy

n(t) = number of shares on record at start of t

m(t+1) = number of new shares sold during and at the ex-dividend price p(t+1)

n(t+1) = n(t)+m(t+1)

V(t) = n(t)p(t)

$D(t) = n(t)d(t)$

We can revise 2 as

$$v(t) = \frac{1}{1+\rho(t)} [D(t)+n(t)p(t+1)]$$

$$= \frac{1}{1+\rho(t)} [D(t)+V(t+1)-m(t+1)p(t+1)] \qquad 3$$

- 3 possible routes by which current dividends might affect the current market value of the firm V(t) or the price of its shares p(t).

- Dividends will affect the value of the firm v(t) via the first term in brackets D(t). In principle, they might also have an impact on the second term V(t+1), but only if V(t+1) were a function of future dividend policy and if D(t) served to convey information about future dividend policy.

- They assume future dividend policy as known and given.

Finally, current dividends can influence v(t) through the third term -m(t+1) p(t+1) - the value of shares sold.

If I(t) is the given level of investment
if x(t) is the firm's total net profit for the period

we know the amount of outside funding required is

$$m(t+1)p(t+1) = I(t) - [X(t)-D(t)] \qquad 4$$

Substituting 4 into 3, we get a cancelling out of D(t)

$$V(t) \equiv n(t)p(t) = \frac{1}{1+\rho(t)} [X(t)-I(t)+V(t+1)] \qquad 5$$

Since D(t) does not appear and since x(t), I(t), v(t+1) and ρ(t) are all independent of D(t) either by their nature or by assumption - the current value of the firm must be independent of the current dividend decision.

What does the market really capitalize?

They distinguish four different approaches:

1. The discounted cash flow approach
2. The current earnings plus future investment opportunities approach

3. The stream of dividends approach
4. The stream of earnings approach

Assume a constant discount rate $\rho(t) = \rho$ for all t. Setting t=0 we can write

$$V(o) = \frac{1}{1+\rho}[X(o)-I(o)] + \frac{1}{(1+\rho)} V(1) \qquad 6$$

We can keep substituting up to an arbitrary time period t

$$V(o) = \sum_{t=o}^{T-1} \frac{1}{(1+\rho)^{t+1}} \Big[X(t) - I(t)\Big] + \frac{1}{(1+\rho)^{T}} V(T) \qquad 7$$

in general the last term $\rightarrow$o as T approaches infinity.

(7) can be written as

$$V(o) = \lim_{T\to\infty} \sum_{t=o}^{T-1} \frac{1}{(1+\rho)^{t+1}}[X(t) - I(t)] \qquad 8$$

which we can abbreviate to

$$V(o) = \sum_{t=o}^{\infty} \frac{1}{(1+\rho)^{t+1}}[X(t) - I(t)] \qquad 9$$

The discounted cash flow approach

Approach taken in capital budgeting. Equivalent to valuing the firm as

$$V(o) = \sum_{t=o}^{T-1} \frac{1}{(1+\rho)^{t+1}}[R(t) - \phi(t)] + \frac{1}{(1+\rho)^{T}} V(T) \qquad 10$$

where R(t) represents the stream of cash receipts and $\phi(t)$ the outlays, abbreviating as before

$$V(o) = \sum_{t=o}^{\infty} \frac{1}{(1+\rho)^{t+1}}[R(t) - \phi(t)] \qquad 11$$

We also know that [x(t)-I(t)]=[R(t)-ϕ(t)] since x(t) differs from R(t) and I(t) differs from o(t) merely by the cost of goods sold (and also by depreciation expense if we wish to interpret X(t) and I(t) as net rather than gross profits and investment. Hence 11 is equivalent to 9.

The investment opportunities approach

Value depends on the value to the owner of the rate of return he can achieve on the investment. At t, the firm invests I(t) the investment projects produce net profits at a constant rate ρ*(t) per cent of I(t) in each period.

The value of this will be

$$I(t)\frac{\rho^{x}(t)}{\rho} - I(t) = I(t)\left[\frac{\rho^{x}(t)-\rho}{\rho}\right]$$

The present worth of this now future goodwill is

$$I(t)\left[\frac{\rho^{x}(t)-\rho}{\rho}\right](1+\rho)^{-(t+1)}$$

and the present value of all such future opportunities is

$$\sum_{t=o}^{\infty} I(t)\frac{\rho^{x}(t)-\rho}{\rho}(1+\rho)^{-(t+1)}$$

Adding in the present value of the uniform perpetual earnings X(o) on the assets currently held, we get an expression for the value of the firm:

$$V(o) = \frac{X(o)}{\rho} + \sum_{t=o}^{\infty} I(t)\frac{\rho^{x}(t)-\rho}{\rho}(1+\rho)^{-(t+1)} \qquad 12$$

To show that the same formula can be derived from 9, note that their definition of $\rho^{x}(t)$ implies the following relation between the X(t):

$X(1) = x(o) + \rho^{x}(o)I(o)$

....

$X(t) = X(t-1) + \rho^{x}(t-1)I(t-1)$

and of successive substitution

$$X(t) = X(o) + \sum_{\tau=0}^{t-1} \rho^*(\tau) I(\tau)$$

t=1, 2, ...∞

Substituting the last expression for X(t) in 9 yields

$$V(o) = [X(o) - I(o)](1+\rho)^{-1}$$

$$+\sum_{t=1}^{\infty}[X(o) + \sum_{\tau=o}^{t-1} \rho^*(\tau) I(\tau) - I(t)](1+\rho)^{-(t+1)}$$

$$= X(o)\sum_{t=1}^{\infty}(1+\rho)^{-t} - I(o)(1+\rho)^{-1} + \sum_{t=1}^{\infty}\left[\sum_{\tau=o}^{t-1} \rho^*(\tau) I(\tau) - I(t)](1+\rho)^{-(t+1)}\right]$$

$$= X(o)\sum_{t=1}^{\infty}(1+\rho)^{-t} + \sum_{t=1}^{\infty}\left[\sum_{\tau=o}^{t-1} \rho^*(\tau) I(\tau) - I(t-1)(1+\rho)\right] x (1+\rho)^{-(t+1)}$$

The first expression is simply a geometric progression summing to X(o)/ρ which is the first term of 12. To simplify the second expression, note that it can be written as

$$\sum_{t=o}^{\infty} I(t)\left[\rho^*(t) \sum_{t=t+2}^{\infty}(1+\rho)^{-t} - (1+\rho)^{-(t+1)}\right]$$

Evaluating the summation in brackets gives

$$\sum_{t=o}^{\infty} I(t)\left[\rho^*(t)\frac{(1+\rho)^{-t+1}}{\rho} - (1+\rho)^{-(t+1)}\right]$$

$$= \sum_{t=o}^{\infty} I(t)\left[\frac{\rho^*(t) - \rho)}{\rho}\right](1+\rho)^{-(t+1)}$$

which is the second term of 12.

Expression (12) suggests that for growth stocks ρx(t)>ρ to be a glamorous stock. If ρX(t)<ρ investment in assets by a firm will reduce its share price - this stresses the importance of the cost of capital.

The stream of dividends approach

Worth of a share is

$$p(t) = \sum_{\tau=o}^{\infty} \frac{d(t+\tau)}{(1+\rho)^{\tau+1)}} \qquad 13$$

in terms of total market share

$$V(t) = \sum_{\tau=o}^{\infty} \frac{D+(t+\tau)}{(1+\rho)^{\tau+1}} \qquad 14$$

M&M demonstrate that 14 is equivalent to 9.

The stream of earnings approach

The capital to be raised in any future period is I(t) and its opportunity cost is ρ. The current value of the firm under the earnings approach is

$$V(o) = \sum_{t=o}^{\infty} \frac{1}{(1+\rho)^{t+1}} [X(t) - \sum_{t=o}^{t} \rho I(t)] \qquad 18$$

18 can be re-written as

$$V(o) = \sum_{t=o}^{\infty} \frac{1}{(1+\rho)^{t+1}} X(t) - \sum_{t=o}^{\infty} \Big(\sum_{\tau=t}^{\infty} \frac{\rho I(t)}{(1+\rho)^{\tau+1}} \Big)$$

$$= \sum_{t=o}^{\infty} \frac{1}{(1+\rho)^{t+1}} X(t) - \sum_{t=0}^{\infty} \frac{1}{\left(1+\rho\right)^{t+1}} \qquad 19$$

$$x \Big(\sum_{\tau=o}^{\infty} \frac{\rho I(t)}{(1+\rho)^{\tau+1}} \Big)$$

Since the last enclosed summation reduces to I(t), the expression in turn reduces to simply

$$V(o) = \sum_{t=o}^{\infty} \frac{1}{(1+\rho)^{t+1}} [X(t) - I(t)] \qquad 20$$

which is the same as expression 9.