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CAPITAL RATIONING: *n* AUTHORS IN SEARCH OF A PLOT

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"When you fall into a man's conversation, the first thing you should consider is, whether he has a greater inclination to hear you or that you should hear him." [21]

I. INTRODUCTION

THE CAPITAL INVESTMENT DECISION occupies a central position in the corporate finance literature, both because of the importance of investment decisions to firms and because of the potential contribution that analysis can make to the quality of decisions in practice. Encompassed within the subject area of capital budgeting is a topic which has been given increasing attention in the literature, in no small part because of controversies over methods of analysis and interpretation. This topic is commonly called capital rationing, although what is understood by that term is far from unambiguous.

The controversies have been less concerned with the empirical existence of capital rationing as a market phenomenon than with optimal decision rules under capital rationing. Joel Dean's original analysis [4] while occasionally still repeated in textbooks, has largely been supplanted by more rigorous analyses employing the mathematical programming apparatus and related theorems. The series of controversies may be traced back to numerous and important differences in the assumptions made by various authors about the phenomenon of capital rationing and within the models they have employed for their analyses. It is the purpose of this article to lay bare the assumptions made and their diverse consequences for decision rules. This task should not be regarded as purely a scholastic undertaking. Finance journals have published many papers on the subject that have nourished misunderstandings and fostered a general misdirection of effort of academic writers on the subject.[1]

In the usual mathematical programming formulation of capital rationing, the

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1. Specific articles, given in the References, are discussed in detail later in the paper.

cash flows from investments are discounted and the objective is to maximize the total present value from investment subject to constraints on capital expenditure (or variants of this). Major attention has been focussed, but in an unsatisfactory way, on two aspects of this problem, which will be the central topic of this paper. First, what discount rate should be used in computing present values and what does this discount rate stand for? Alternatively, what should be the criterion for optimization? Second, if the constraints on expenditure are binding for a given firm, has the discount rate measured the firm's opportunity cost of capital properly, or is there, alternatively, a discount rate which "clears the market"—the internal demand for funds and the externally made available funds?

In dealing with these issues, most authors have made a series of additional assumptions in an attempt to strip away all but the essentials of the framework for purposes of analysis. Among these are a linear function for utility of consumption and constant returns to scale on investment.[2] Unhappily, these assumptions have contributed to the confusion rather than to a general clarification of the issues. Their implications are dealt with here so as to make clear the need to avoid such "simplifications."

In the present discussion, capital rationing will be interpreted in the way most participants in the controversies have done—as a market-imposed limitation on the expenditures a firm may make. This is not the interpretation I have made in the past nor the one I wish to pursue in the future. Indeed, the point of departure for my first work on the subject [17] was the piece by Lorie and Savage, "Three Problems in Rationing Capital" [12]. A reading of that article can leave little doubt as to its focus on the problem *managers* face in the allocation of resources to capital projects, and my own contributions have been directed at utilizing the information content of the programming formulation as an aid to decision making and not as a positive theory of financial markets.

The analyses offered here carefully lay out the economic assumptions, case by case, for covering the important aspects of external capital rationing. Accordingly, here the initial model will be concerned with "pure capital rationing" in which both the firm and its owner are simultaneously limited in their access to financial markets. The more commonly treated situation, that which deals with the corporate practice of operating within capital budgets, is one in which the firm exists apart from its owners. There may be many owners, and (initially) only the firm is supposed to be limited in the amount of funds it can obtain from financial markets. This issue is analyzed next. The simplifying assumptions of Baumol and Quandt [1], those of linear utility functions and constant returns to scale are next shown to be sources of a number of the difficulties in which subsequent writers became enmeshed. The search for the elusive discount rates is assessed before closing with comments on the market phenomenon of capital rationing.

The conclusion of the present analysis is that the interpretation of capital rationing as a market phenomenon is inconsistent with its internal assumptions and its consequences are at variance with observation. Although this conclusion may not be a surprise to some, prominent and apparently persuasive writers[3] have

2. Additionally, post-horizon cash flows are almost always ignored.

3. For example, Baumol and Quandt [1].

published elaborate discourses to the contrary. Hence, mere assertion of such a proposition has not, and most likely will not stem the flow of publications on this subject, and a careful dissection of the arguments is still in order.

Perhaps mathematical programming possesses its own seductive appeal. Alternatively, it may be that concentration on the mathematics is responsible for obscuring the underlying issues that are being treated. Therefore, to avoid this pitfall, the present discussion will eschew entirely the use of mathematical programming. Rather, a graphical two-period Fisherian apparatus will be employed throughout this paper. By this means, it may be hoped, all the issues may be successfully exposed, the pitfalls and fallacies uncovered, some further insights may be gained, and future writers may be freed to direct their attention to more meaningful problems.

II. "Pure" Capital Rationing

The so-called "pure" capital rationing problem[4] is supposed to exist when an owner-firm is constrained in his investment programs by externally imposed, explicitly stated sums for the foreseeable future, and thus the entrepreneur must make his capital expenditure decisions accordingly. The empirical existence of pure capital rationing is not at issue here. Although they deal almost exclusively with this case in their models, Baumol and Quandt's introductory paragraph suggests that, as a market phenomenon, it is likely to be temporary. The argument that capital budgeting is inaccurately described in this way, but rather represents a partial solution to organizational as well as economic problems, was presented by J. Hirshleifer [9] and by the author [18] and will not be repeated here. However, attempts at disposing of the criterion problem under pure capital rationing appear to have failed, witness the literature, and so it must be dealt with once again.

At issue in this debate is the role of market discount rates in the optimal allocation of funds under capital rationing, and the status of the "Separation Theorem" in Fisherian analysis which applies in the perfect capital market case and according to which investment decisions of a firm may be made independently of the time-preference for consumption by its owner. Without the separation property much of what is taught on capital budgeting would go out the window. (In what follows use is made of the familiar two-period consumption investment decision under certainty framework developed by Irving Fisher [7] and elaborated by Hirshleifer [8] and treated at length in a number of recent texts [9, 6]. Some familiarity with this framework is assumed in order to proceed more directly to the issues of interest.)

In the two-period world portrayed in Figure 1, the resources of the owner-firm giving rise to consumption opportunities in periods 0 and 1 are displayed along $\hat{K}_0 A \hat{K}_1$, labeled "Productive Opportunity Line."[5] In the absence of access to

4. Called, "hard rationing" by Carleton in [3].

5. Actually, these are the "undominated" consumption opportunities in the sense that with any given amount of current consumption, no higher consumption next period is possible with these resources, and similarly, with any given amount of next period consumption, no higher current consumption is possible. Thus, the Productive Opportunity Line is seen to be the result of an optimization, for given resources.

financial markets, the owner would choose an amount of investment so as to attain the most preferred consumption vector. He does this by finding the point on the Productive Opportunity Line which reaches the highest indifference curve representing his consumption preferences, here point C, which yields consumption amounts C_0 and C_1 in periods 0 and 1, respectively.

In the presence of perfect capital markets, in which borrowing and lending can take place without limits (so long as a loan can be repaid) at the market rate of interest, r, the owner's consumption opportunities are extended to all points along the line $K_0^{***}AK_1^{***}$. This line, denoted the "Financial Market Line" displays all amounts of consumption achievable in conjunction with the given Productive Opportunity Line either by borrowing or lending at interest rate r. With these additional opportunities, the owner whose preferences are as portrayed in the figure will choose the consumption vector (C_0^*, C_1^*) at point C^*, which he prefers to point C. He achieves point C^* by investing an amount $\hat{K}_0 - A'$ in productive assets which yield an amount A'' in period 1. By borrowing in the financial markets an

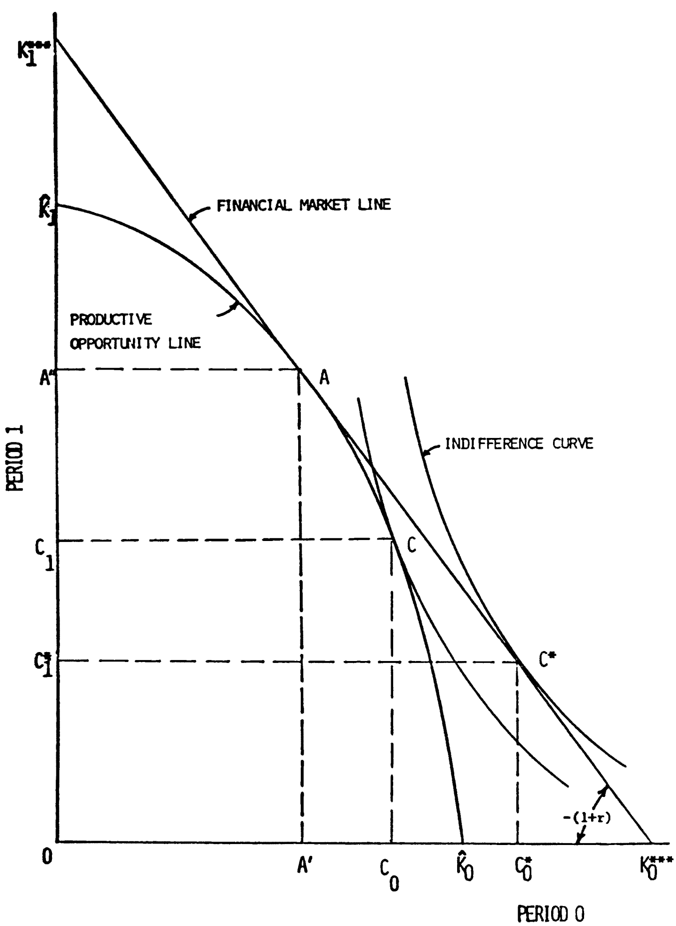

Figure 1. Fisherian Two-Period Investment Analysis Perfect Capital Markets

amount $C_0^* - A'$ and repaying in period 1 an amount $A'' - C_1^*$ the owner attains his preferred consumption point C^*.

The optimal amount of investment in productive opportunities is determined by locating the point of tangency between the Productive Opportunity line and the Financial Market line, point A. At this point the marginal productivity of capital is equal to one plus the interest rate. This point also has the property that it has the maximum present value of all points on the Productive Opportunity Line at the market interest rate r. Point K_0^{***} on the horizontal axis locates the present value of point A. Simultaneously point A also has the maximum terminal value of productive opportunities, which value is indicated by point K_1^{***} on the vertical axis. The preferred consumption point, C^*, is characterized by equality of the marginal time preference of the owner with the market rate of interest (equality of the slopes of the indifference curve and the Financial Market Line). In terms of the diagram, the investment decision therefore depends only on the location and shape of the Productive Opportunity Line and the interest rate, and not on the shape of the indifference map.

A few comments about this solution to the investment problem are in order. First, the model holds strictly only for the owner of the Productive Opportunity Line—the opportunity for earning more than the competitive return on capital.[6] As we shall show, below, the conclusions are not altered if many consumers share the ownership of the Productive Opportunity Line. Nor does it matter for purposes of decision rules or characterizations of optimal points, in the perfect capital market case, whether these consumers also own (i.e., derive income) from other financial or productive assets. This latter conclusion does not hold in the case of pure capital rationing.

In the managerial process of capital budgeting frequently limits are set on expenditures on capital account (or more, accurately, on *plans* for such expenditures). Those writers modelling "pure" capital rationing instead set limits on expenditures equal to internally generated funds plus pre-set limits (possibly zero) on external financing which are independent of the profitability of proposed investments.

This situation is depicted in Figure 2, in which a binding borrowing limit in the amount of $B' - A'$ has been superimposed on the productive and financial opportunities of Figure 1. The highest relevant Financial Market Line now ends at B, the point at which the maximum permissible amount of borrowing is combined with production at point A. The borrowing limit has eliminated points along the dotted line BK_0^{***} from the consumption opportunity set. Nevertheless, the set of consumption opportunities does not end at B. Instead, a new locus of consumption opportunities is created in the following way. For each point on the Productive Opportunity Line below point A it is supposed that an amount of borrowing equal to the maximum, viz., $B' - A'$, takes place. The rate of interest is assumed to be r, and so a family of Financial Market Line segments begins at each point of the Productive Opportunity Line below A and terminates at the curve BE. (Two dotted

6. This excess return is usually called economic rent (and the discounted stream of rents is sometimes called "goodwill"). Of importance for later discussion is that the Productive Opportunity Line is itself the result of an optimization, in which "projects" of infinitesimal scale have been arrayed in order of decreasing returns.

lines, XY and GE exemplify this.) Curve BE depicts consumption opportunities generated by combining various amounts of investment in productive opportunities and maximal borrowing in the financial market. Thus the entire set of consumption opportunities is bounded by line $K_1^{***}ABE$.[7]

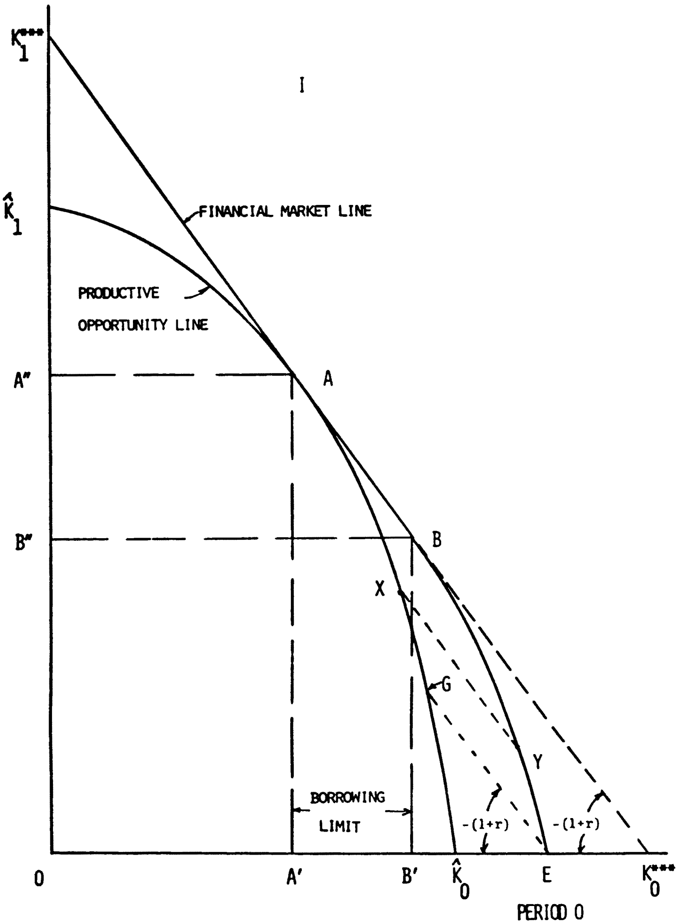

FIGURE 2. FISHERIAN TWO-PERIOD INVESTMENT ANALYSIS "PURE RATIONING"
I

Figure 3 superimposes the indifference map of Figure 1 onto the new locus of consumption opportunities of Figure 2. It may first be observed that point B is not the optimal consumption point for the individual whose consumption preferences have been portrayed in the figure. Quite clearly, there exist many points along BE which may be preferred to point B, and of these, point C' is optimal in that it permits the owner-consumer to reach his highest indifference curve, given the borrowing limit. To attain it, he invests in productive opportunities an amount sufficient to reach point D on the Productive Opportunity Line, then he rearranges

7. For example, producing at point X allows consumption at point Y by borrowing an amount $C_0^Y - K_0^X = B' - A'$, where C_0^Y is the abscissa of point Y and K_0^X is the abscissa of point X.

his income stream to achieve the consumption vector (C_0', C_1') at point C'. At this point indifference curve I_2 is tangent to the "Consumption Possibilities Line" $K_1^{***}ABE$.

Certain important implications may be derived from this figure. First, in the face of a borrowing constraint, the investment decision cannot be made independently of the consumption decision: the Separation Theorem fails to hold. If the borrowing limit is *not* binding, the optimal amount of investment is still the amount at which the marginal return on investment equals the market interest rate and is also the point at which the discounted value of productive opportunities at the market interest rate is maximized, although this "present" value is not realizable by the owner. It would be incorrect to state, however, that either or both of these conditions *determines* the optimal amount of investment since, even without a binding borrowing limit, the fact that it was not binding had first to be determined.

When the borrowing constraint is binding, the marginal return on the optimal amount of investment is greater than the market rate of interest. Further, in both

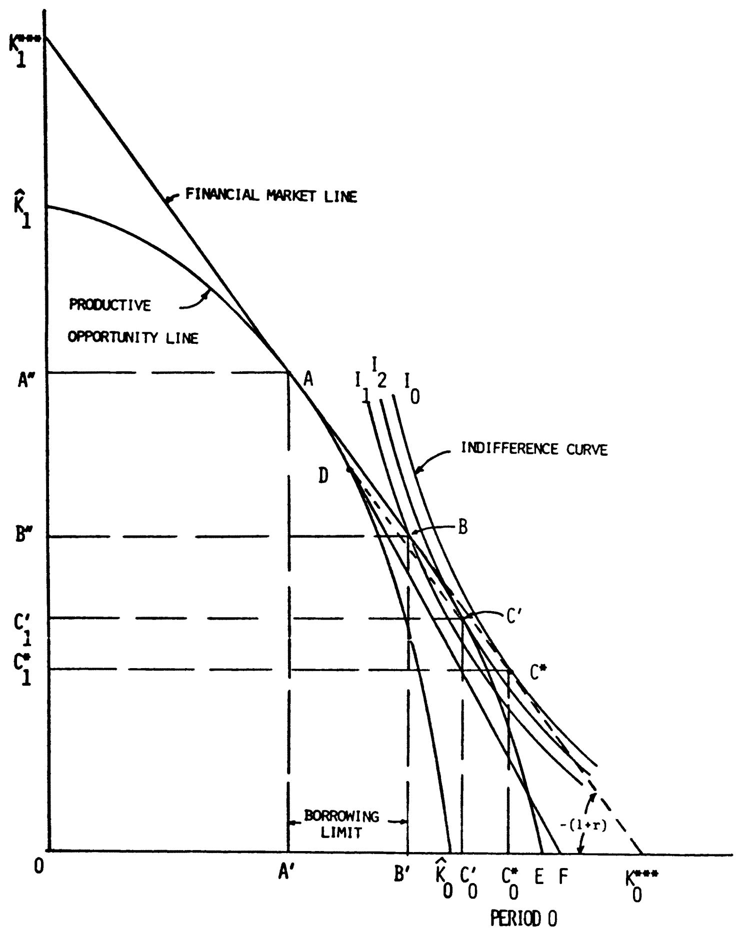

FIGURE 3. FISHERIAN TWO-PERIOD INVESTMENT ANALYSIS "PURE RATIONING" II

instances, optimal consumption is characterized by points at which the marginal rate of time preference is higher than the interest rate; in fact, it is equal to the marginal return on investment.[8] The marginal rate of return on investment at every point is given by the slope of the tangent to the Productive Opportunity Line at that point. In Figure 3 the tangent has been drawn at point D, which corresponds to the optimal consumption point C'. As indicated in the figure, this tangent intersects the Period 0 axis at point F, which represents the maximum discounted value of productive opportunities using a discount rate r'.

The rate r' is the marginal rate of return on productive investment at point D. By construction, the slope of the tangent at D is the same as the slope of the tangent to curve BE at the optimum consumption point C'. Discounting consumption at C' at the rate r' would therefore yield the highest discounted value of consumption available when discounting is done at rate r'. It should be noted, however, that no optimization procedure may be applied which employs this discount rate in its objective function because the value r' will not yet have been determined. It would also be inappropriate to call these discounted values *present values* since they are the result of an arbitrary discounting process, and not amounts realizable in the present by transactions using available market interest rates.

The above analysis is not significantly altered for the firm which is owned by many individuals, each of whom has all his assets in the form of shares of this firm. It does become necessary now to make a distinction between the firm and its owners. In Figure 4 the Productive Opportunity Line is extended into the second quadrant. Here the firm is assumed to be able to operate without additional funds. However, as the point of tangency in the figure between the Financial Market Line and the Productive Opportunity Line at Point A indicates, borrowing is required for the optimal level of output.[9]

Although with a single owner it would not be necessary to do so, it is possible to interpret this figure to bring out the distinction between the owner and the firm in the following way. Under certainty and perfect capital markets, a single interest rate prevails for productive and consumptive purposes. A family of Financial Market Lines, with slope given by the interest rate, is used to locate the optimum amount of investment, and the previously stated perfect market investment rules apply: invest until the marginal rate of return on capital equals the interest rate. In this figure, however, the amount invested may be factored into two parts: amount $\hat{V}_0$ on private account (forgoing current consumption which could be obtained by liquidating the firm now) and amount A' on "corporate account." The amount A' on corporate account is repaid with interest at the rate r, which is portrayed by the distance on the vertical axis between A'' and V_1.[10] V_1 then represents the terminal value of the consumption possibilities generated by the optimal amount of invest-

8. This conclusion becomes obvious when the maximum amount of borrowing is zero and curve $K_1^{***}ABE$ collapses onto $K_1^{***}A\hat{K}_0$.

9. It would have been plausible to *start* the Productive Opportunity Line at the origin, as some writers (e.g., [6] have done. However, the intersection of the Productive Opportunity Line with the horizontal axis may be thought of as the liquidation value of the firm, and that some production (i.e., future consumption possibilities) may be generated without additional resources.

10. A little geometric reasoning will show that $A'' - V_1 = (1+r)A'$.

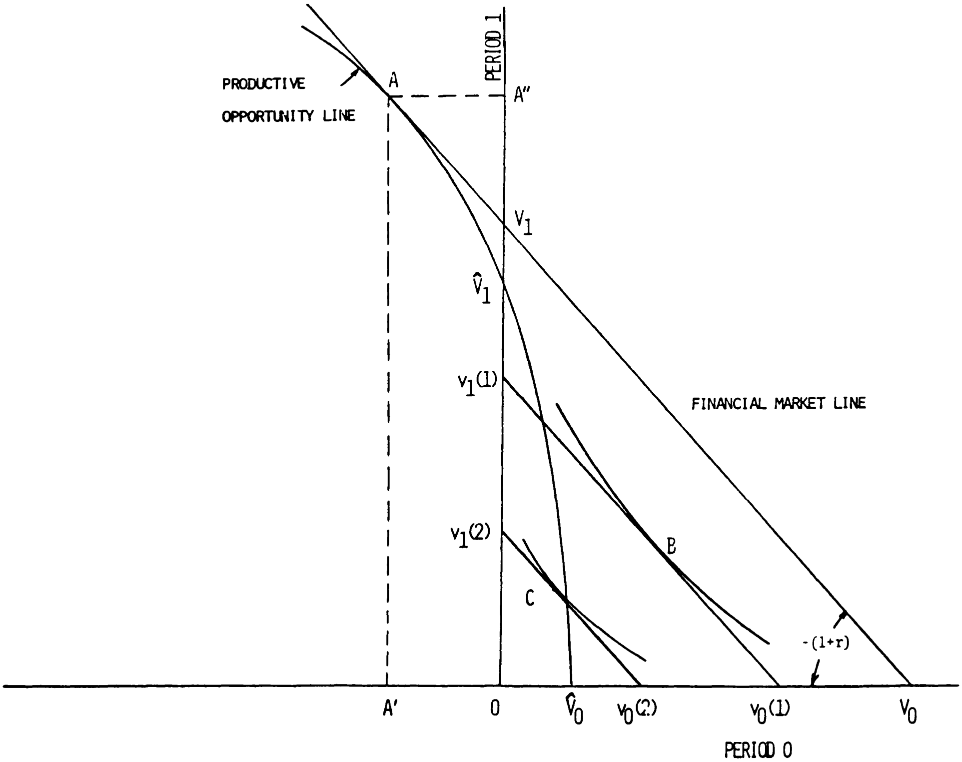

FIGURE 4. FISHERIAN TWO-PERIOD INVESTMENT ANALYSIS: MULTIPLE OWNERS

ment in the opportunities represented by the Productive Opportunity Line possessed by the owner. The owner separately rearranges his consumption to suit his preferences by "trading" part of V_1 for some consumption in Period 0.

Once it is observed that the terminal value V_1 is available to be redistributed among the two periods, it is also clear that this amount is available for distribution among the several owners, if there are more than one. In the figure this has been depicted by two owners, one of whom is assumed to own two-thirds of the firm, the other the remaining one-third. A separate Financial Market Line is drawn for each of the two owners, one between $v_0(1)$ and $v_1(1)$, representing the present and terminal values of the first owner's share of the firm's proceeds, respectively, and between $v_0(2)$ and $v_1(2)$ for the second owner. The figure is drawn so that both $v_0(1)+v_0(2)=V_0$ and $v_1(1)+v_1(2)=V_1$.

The two owners' respective choices for dividing their consumption between the two periods is indicated by points *B* and *C* respectively. Each of the owners arrives at his optimal point by reference only to the amount available (viz., $v_1(1)$ and $v_1(2)$, respectively, or $v_0(1)$ and $v_0(2)$, the corresponding present values) and to the interest rate, the two parameters which, for each owner, determine the location of that Financial Market Line relevant for him.

In sum, the Separation Theorem holds and, indeed, gains its importance from this case. The optimal investment decision is made in the usual way, by whoever is the agent for the owners: invest until the marginal rate of return equals the interest rate, or, maximize the present value of investment computed at the market rate of interest.

If, in this perfect capital market case, the owners have claims to income from other sources, they still would not want this firm to alter its investment decision rule. It would be inappropriate, in that case, to draw an indifference map for each owner's consumption preferences from *this* source of income. Instead, the present value of all claims may be added, and the optimal consumption decision for each recipient of income is still consistent with the principle of equating marginal time preference with the market interest rate.

III. Capital Rationing for the Firm

Having made a distinction between the firm and its owners, it is now possible to analyze the situation usually assumed under the notion of "pure" capital rationing, in which the *firm* is constrained in the amount of capital it can obtain. In Figure 5, which parallels Figure 4, the perfect capital markets solution (point A) is assumed to be unavailable because of a borrowing limit *on corporate account,* in the amount of B'. The optimal amount of investment, given the Productive Opportunity Line as drawn, requires borrowing the maximum amount and achieving point B for the

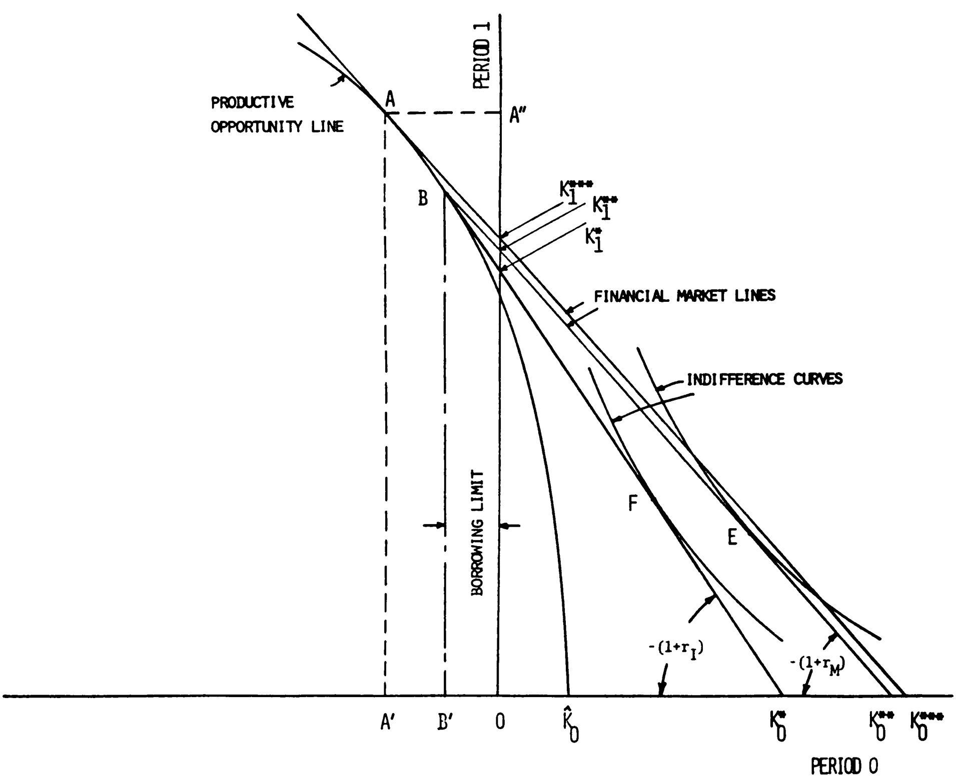

Figure 5. Capital Rationing for the Firm

firm. As the figure shows, the maximum terminal value available for distribution among the owners, after repayment of corporate debt at the interest rate (here denoted by r_M), is now K_1^{**}. While this is less than K_1^{***}, the value achievable under perfect capital markets, it is the best the owners can obtain under the assumed circumstances.[11]

If the owners of the firm are personally able to borrow and/or lend in the financial markets at the same interest rate, r_M, they will make their respective consumption decisions based on this rate. As in the perfect capital markets case, they will combine the present values of their incomes from all sources and determine the preferred temporal rearrangement of consumption solely by reference to the total present value and the interest rate.

In Figure 5, assuming a single owner with no other assets, the preferred consumption point is located at E, the point of tangency between an indifference curve and a Financial Market Line through point K_1^{**} (and also through point B). The decision rule which the firm has followed in this case is to maximize the net present value of investment at the market interest rate, subject to the corporate borrowing limit. The firm was able to arrive at the optimal amount of investment without reference to the tastes and preferences of its owners, or on knowledge of the amounts of the owners' income from other sources.

An objection may be validly raised with respect to this solution. Given that the owners of the firm are able to borrow and lend in unlimited amounts at the rate r_M, they can improve on the solution at K_1^{**} by personally *lending* funds to the firm so as to nullify the corporate borrowing limit. If they do so, the point A and value K_1^{***} applicable in the perfect capital markets case obtain here also. Indeed, making the owners into financial intermediaries restores the former solution. In doing so, the realization creeps in that this form of rationing in a certainty framework is artificial and unrealistic. If the capital market were to impose a corporate borrowing limit, it may be because of potential bankruptcy (and limited liability) as the primary reason. Thus, it would be inappropriate to proceed with a veiled uncertainty analysis by stating that a) owners would be willing to lend to the firm where others would not, and b) that owners (perhaps because of their personal unlimited liability) would be able to borrow in unlimited amounts based on the security of their ownership in the rationed firm. In other words, if there is plausiblility to the case of rationing of the firm, there is little plausibility to assume it away by making owners generally the financial intermediaries.[12]

In Figure 5, a tangent was drawn to the Productive Opportunity Line also at the optimal production point B. The slope of this line, which intersects the vertical axis at K_1^* and the horizontal at K_0^* may be interpreted as fixing an interest rate, r_I. It would therefore be accurate to say that point B maximizes the net present value of the Productive Opportunity Line at interest rate r_I without reference to any borrowing limit, and that the marginal productivity of capital is then equal to r_I. In

11. The possibility that the borrowing rate for individuals is less than r_M is explicitly excluded here.

12. This remark is not intended to deny that on some occasions major stockholders become "lenders of last resort" to small or closely-held firms. Explanations for such actions require, however, a wider model encompassing tax aspects, questions of corporate conrol, and possible divergences between expectations for the firm held by lenders and insiders.

fact, r_I is the answer to the question, "What interest rate would make the firm borrow only an amount that leads it to produce at the optimal point B?" Clearly, this is an *internal* rate of return. One must nevertheless wonder why the question was raised at all, since this interest rate is not the one used by any of the owners for the purpose of making optimal consumption decisions.[13] Point F is not optimal for anyone, not even for the single owner whose assets are entirely tied up in this firm, and for whom the earlier model of capital rationing of Figure 3 applies.

IV. Investment with Different Interest Rates on Personal and Corporate Debt

The preceding discussion permits easy resolution of the problem of divergent interest rates between personal and corporate debt; hence, a short digression. In Figure 6, a Corporate Financial Market Line, with the interest rate denoted by r_C is drawn tangent to the same Productive Opportunity Line as before, at point B. In the absence of borrowing limits the firm will borrow an amount B' and repay next period an amount $B'' - K_1^C$ leaving the terminal value K_1^C to be distributed among the owners.

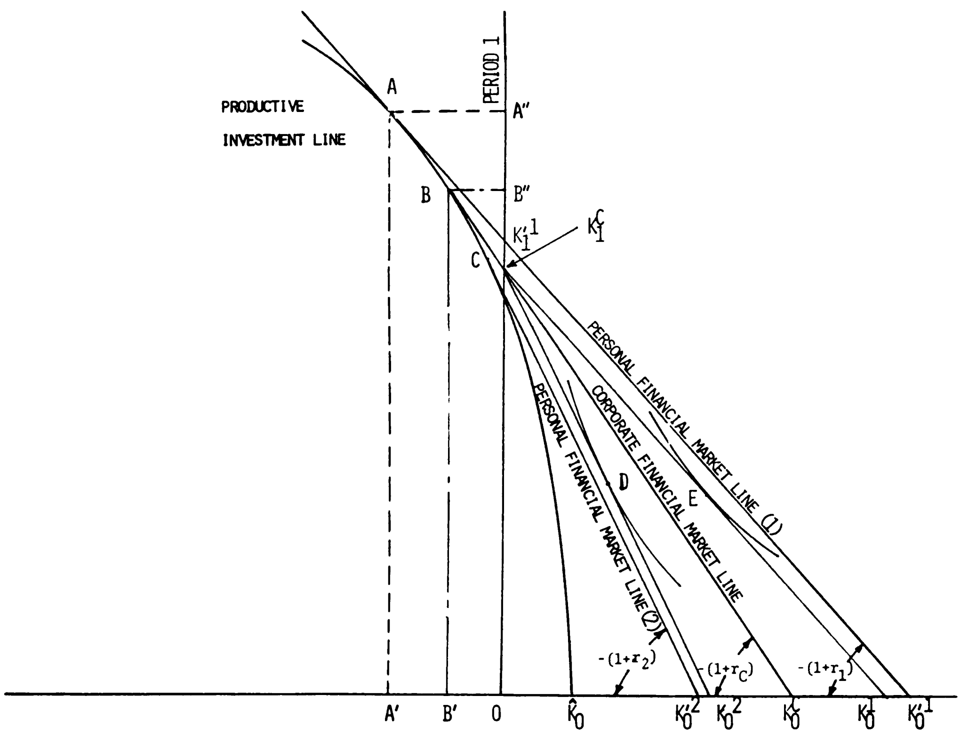

FIGURE 6. CAPITAL INVESTMENT WITH DIFFERENT INTEREST RATES ON PERSONAL AND CORPORATE DEBT

13. Cf. the discussion of the literature, below.

Two different Personal Financial Market Lines are drawn to meet point K_1^C, one using a rate r_1 which is less than the corporate rate, r_C, and one using rate r_2 which is greater.[14] It is assumed that only one of the two personal interest rates, r_1 or r_2 applies as discussed below, and that each applies for both lending and borrowing. If the corporate borrowing-lending rate is higher than that for individuals the possibility that owners will become financial intermediaries once more needs to be considered. More plausibly, if the corporate rate is lower, in which case the steeper of the two Personal Financial Market Lines applies, owners will use terminal value K_1^C and r_2, as well as their other assets, to determine their preferred consumption vector. A single owner with no other income would choose the consumption pair located at point D.

When personal borrowing-lending rate r_2 applies, it is easy to see from the figure that the owners would be better off if the firm were to follow its investment rule of maximizing present value at the corporate interest rate, r_C, and not at the personal rate, r_2. Were it to use r_2 the firm would choose to produce at point C at which Personal Financial Market Line (2) is tangent to the Productive Investment Line. However, since corporate debt is repaid at the interest rate $r_C < r_2$, the terminal value available for the owners would lie below K_1^C on the vertical axis, and this is worth less to the owners.

Were the firm to use the personal interest rate r_1 to make its investment decision (should this be the rate for personal financial transactions), it would produce at point A as a result of maximizing the present value of second period output. However, since corporate borrowing is not possible at rate r_1 but the higher rate r_C must be paid on the corporate loan in the amount of A', the terminal value of consumption in the two periods available for distribution to the owners is also less than K_1^C: a line through A parallel to the Corporate Financial Market Line necessarily lies below that through point B since point B is a point of tangency.

Once more, the conclusion is the same as before. The investment rule for firms which must borrow at a corporate borrowing rate different from the rate for personal borrowing is to maximize their owners' welfare by a) maximizing the present value of investment (output) using the corporate borrowing rate, or b) (which amounts to the same thing) investing until the marginal rate of return on capital equals the corporate borrowing rate.[15] This applies even though owners choose consumption vectors which satisfy the condition that the marginal time preference for consumption is equal to the personal interest rate.[16]

14. The steeper of the two uses r_2 since a given future value has a smaller present value when a higher discount rate is used.

15. In the multiperiod case with continuous productive opportunities the second form of the criterion becomes one of component by component equality of the vectors of returns and cost. See [19].

16. This discussion does not reslove the indeterminacy in certain instances when personal borrowing and lending rates are assumed to be different, presumably with the former, $r_{pb} > r_{pl}$, which denotes the latter. In terms of the distinction between the firm and its owners, as depicted in Figure 6 for example, the divergence between the two personal interest rates becomes relevant only when point B, the tangency point between the Corporate Financial Market Line and the Productive Opportunity Line lies inside the first quadrant. Such a point would not now be optimal since under this assumption, the owners would not want the firm to borrow. Instead, the firm would liquidate some of its assets (at least in amount $0B'$, where B' is now on the positive horizontal axis), and two questions remain to be

V. Review of the Literature

Given the rate of publication on this subject, a comprehensive survey and critique of the literature is neither possible nor so obviously desirable. It would be useful, however, to attempt to review the writings of those authors most frequently cited, and to emphasize those differences which appear to have created a Tower of Babel. It will be instructive at times to refer to parenthetical articles only to illustrate how changes in assumptions or the introduction of assumptions of "convenience" may have had the disasterous consequences of vitiating their conclusions.

The piece which properly focused attention on the criterion problem yet produced most confusion in subsequent writings is that by Baumol and Quandt [1]. Unfortunately, their own discussion was in part at variance with their assumptions and this contributed to the confusion created in the minds of their readers. In the terminology used here, they considered the firm as making investment decisions subject to externally imposed fixed expenditure ceilings in every period up to an unspecified horizon.[17] Investments are assumed to exhibit constant returns to scale, in contrast to those models, e.g., in [17] in which investment is in projects, and where multiple units of projects are assumed to have different payoffs and costs.[18] Market rates are asserted to be irrelevant [p. 322], which implies that the owners of the firm, if there are many, have no access to financial markets.

The Baumol and Quandt reformulation of the model because of deficiencies in the first one[19] is in terms of maximization of utility of cash withdrawals and suggests that there is only one owner, all of whose income is derived from the firm.[20] Their formulation employs a linear utility of consumption function, which

answered. Should more than $0B'$ be liquidated, and should the liquid assets be invested in the corporate financial market or distributed to the owners?

The latter question might be resolved by appeal to an additional assumption, appropriate for a two-period world, that firms engaged in production cannot act as financial intermediaries for their owners simply to gain the advantage of the difference between the corporate and personal lending rates, if the latter is lower. Indeterminacy with respect to how much to liquidate cannot be resolved without reference to the time preferences for consumption by the owners. Indeed, this is the case Hirshleifer analyzed [8], except that additional difficulties are encountered because owners of this firm may have income from other sources, and some owners may want to lend in the current period while others may want to borrow on personal account. This raises all the difficulties for the decision process by the firm which absence of the Separation Principle entails.

17. Although their introductory paragraph suggests that capital rationing is either self-imposed or temporary [p. 317] their models take as given the existence of "hard" rationing up to the horizon. Unlike the author's discussion in [18], rationing here is not a planning or control device of the firm. The last line of [p. 319] does, however, cast doubt on the degree to which they maintained this view.

18. As the discussion below will demonstrate, constant returns are a source of additional difficulties in resolving these very issues.

19. This was essentially that of Lorie and Savage, [12], as stated mathematically by Weingartner [17, Chapter 3] with modifications such as constant returns to scale.

20. This inference is tenuous because on p. 321 there is reference to the possible use of funds for investment outside the firm which, presumably, will generate income that enters into the utility function of the owner.

introduces no difficulties when owners derive income from several sources, since marginal utility for consumption is constant in each period.[21]

Not surprisingly, Baumol and Quandt conclude that, for the rationed owner-firm who has no access to financial markets, the time preferences for consumption of the owner will determine the optimal set of investments. Also, *once the investment decision has been made*, a marginal productivity of capital will have been found which accurately orders the consumption preferences of the individual who made the investment decision, that is, which generate a set of personal discount rates that have this property. In terms of the two-period analysis above, it is a function of the slope of the tangent *DF* in Figure 3.

Authors writing in response to Baumol and Quandt have focused on two distinct issues. The first to be discussed here is the need for a utility function framework and the role of market interest rates for making capital expenditure decisions. The second is the determination of discount rates on the assumption that market rates fail to be of use. Evidently, writers were not as sanguine as Baumol and Quandt who had concluded that, by determining discount rates from ratios of (constant) marginal utilities of consumption between periods they had found a construct that "is usable directly in the computations required for optimal investment project selection" [1, p. 329].

On the first issue, Elton [5] made a singularly unhappy attempt to remedy the situation.[22] He did, however, make plain at the outset that he was concerned with a firm with many owners distinct from the firm, and that by capital rationing he meant that the "firm has no external borrowing opportunities but the firm's stockholders do have access to capital markets" [p. 573]. He concludes that market rates "are important [sic] to a firm operating under capital rationing" [p. 583].[23]

Stewart Myers, in a succinct *Note* [16], reviews the Baumol and Quandt formulation of the Lorie-Savage-Weingartner model and resolves the criterion problem in

21. Contrary to their assertion in footnote 2, p. 326, this assumption in combination with the constant returns to scale assumption poses other difficulties, some of which are dealt with in the paper by Manne [14] and some of which will be discussed below. A later paper by Merville and Tavis [15] applies a linear utility function maximand to this author's Basic Horizon Model [17, Chapter 8] allowing for discrete projects and explicit, limited external borrowing and lending opportunities. They concentrate on divergent borrowing and lending rates, presumably applied to both individuals and firms. [Their geometric illustrations are correct if, on line 2 of p. 112, the term L_0H_0 is replaced by L_0H_0''.] The dependence of their model on utility of consumption *or* dividends [p. 113] (assumed to be equivalent?) is appropriate only if owners, as well as firms, are rationed, as was shown above. Their objection to maximizing terminal value is unfounded if owners are not rationed while the firm is. Their "generalization" unfortunately also does not solve any managerial problems.

22. Elton's piece unfortunately contains numerous errors and inaccurate representations of the positions of other writers. For example, in his citations of this author's work [5, pp. 573, 581 and 583], and of Baumol and Quandt [5, p. 582] the assertions he claims were made are simply not to be found. His interpretation of the fundamental paper by Hirshleifer [8] is also almost wholly incorrect.

23. Elton is unique in interpreting the indifference curves of Fisherian analysis to be derived from "preferences of dividends from a particular firm" [5, n. 7, p. 576]. It is difficult to imagine a meaning for such a concept unless it refers to the unusual, but actual restriction placed on trustees by some investors who proscribe investment in the securities of firms selling alcohol or tobacco, or who do business in the Republic of South Africa.

favor of the latter[24] by showing the equivalence of the Baumol and Quandt utility formulation and the modified Lorie-Savage-Weingartner form. To do so he employs the argument that investors adjust their investments such that the marginal rate of time preference equals the market interest rate.[25] Quite clearly, without going back to a mathematical programming formulation, this case was covered in the discussion relative to Figure 5, above, in which investors are assumed to have access to borrowing and lending opportunities limited only by their ability to repay. To return the compliment: Myers is right—but for a case different from that apparently covered by Baumol and Quandt. As was shown above, when owners have access to financial markets but firms either have limited or no access to them, firms which maximize their owners' economic welfare will maximize the present value (or terminal value) of their investments at the market interest rate subject to their borrowing constraints.[26]

VI. Constant Returns to Scale

Many authors writing subsequently to the initial work on programming methods applied to capital budgeting dropped the notion of a project and assumed instead constant returns to scale without qualification.[27] This assumption of convenience was thought to be inocuous [1, pp. 320–21] but proves otherwise.[28] Perhaps it was also motivated by the conclusion, subscribed to by a number of economists, that long run average cost curves are flat over a large range of plant sizes[29] [11]. Because it is also combined with the assumption of linear utility functions which, it will be shown, also leads to peculiar and unwarranted conclusions for the more general case, it is worth looking at this case in somewhat greater detail.

In terms of the two-period world utilized throughout this paper, investment alternatives exhibiting constant returns to scale would be portrayed as negatively sloped straight lines, emanating from the point on the horizontal axis representing the owner's current capital, point $\hat{K}_0$ in Figure 7. In the figure these are projected only to the vertical axis, which once more represents consumption in the next period.[30] For the moment there is no need to extend these lines into the fourth quadrant, as was done in Figures 4, 5 and 6.

24. "Weingartner was right in the first place; the 'Hirshleifer Problem' does not apply to corporate capital budgeting decisions" [16, p. 92].

25. [16, p. 91]. Myers does not say so explicitly, but appears to assume that investors make this adjustment not out of whimsy but because they have access to markets in which they can trade off consumption in one period for consumption in another at given market interest rates.

26. If the corporate and personal interest rates are different, the firm's decision rule should be to maximize the present or terminal value of investment at the corporate interest rate, subject to the borrowing constraints.

27. See [1, 2, 3, 13, 14, 16] among others.

28. Some of the difficulties with the information content of the dual variables in the constant returns to scale case were discussed in [17, Ch. 4].

29. It is important, of course, to distinguish between empirical findings, and the *ex ante* range of alternatives considered by firms in their capital investment decisions.

30. Constant returns without limit at a rate of return higher than the market rate is obviously inconsistent with perfect capital markets in which any individual borrower (investor) is assumed to be unable to affect the market price by his single transaction.

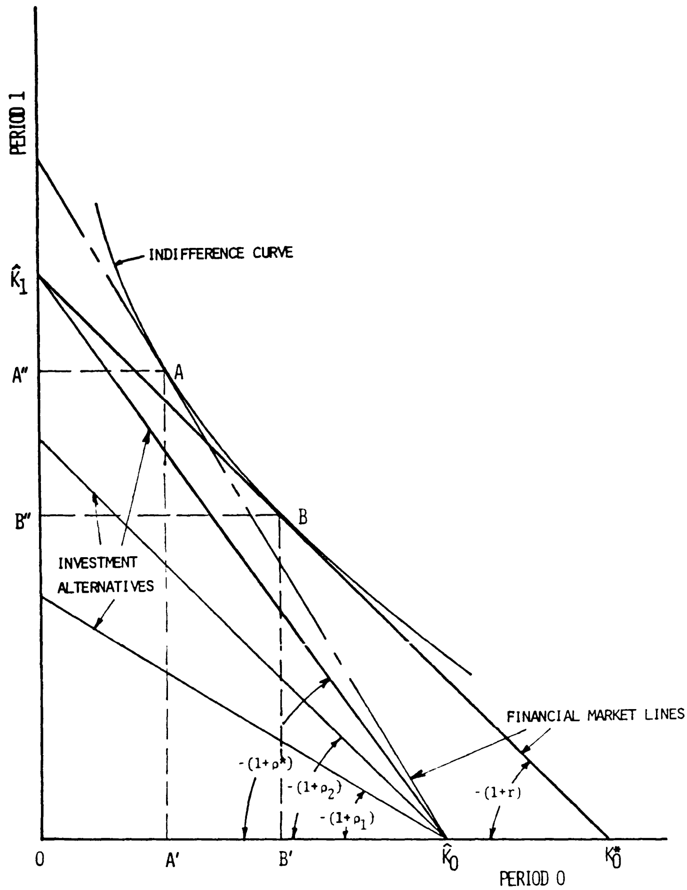

FIGURE 7. FISHERIAN TWO-PERIOD ANALYSIS CONSTANT RETURNS TO SCALE AND PERFECT CAPITAL MARKETS

Among the alternative investments only one (if any) will be selected: that with the highest marginal (and average) rate of return, shown in the figure as having the steepest slope. (Rates of return there are denoted by ρ_1, $\rho_2, \ldots$, and the highest rate of return is denoted by ρ^*.) The interest rate applicable to purely financial transactions is once more denoted by r, and the Financial Market Line has slope $-(1+r)$. Assuming perfect capital markets, the optimal amount of investment as well as the choice between borrowing and lending is determined by the relationship between r and ρ^*. For the moment the indifference map of the owners is assumed to have the usual concave shape.

If the rate of return on the optimal investment alternative is greater than the market interest rate, the investment will be pushed to its limit, and the owner(s) will borrow in the financial market to support their current consumption. Alternatively, if the market interest rate exceeds the most favorable return available to owners of this firm, they will choose to consume some of their capital and to provide for next period's consumption by lending in the present: the firm will not operate, i.e., produce output the next period. With the exception of the case in which $r = \rho^*$

(called degeneracy in programming terminology) the optimal scale of investment either is zero or the maximum possible.[31]

In Figure 7 this is shown by two financial market lines, with slopes corresponding to the two cases mentioned. With $r < \rho^*$ the Financial Market Line relevant for consumption decisions goes through K_0^* and $\hat{K}_1$ and lies entirely above the investment line $\hat{K}_0\hat{K}_1$ except at point $\hat{K}_1$. The optimal amount of investment is therefore the entire amount of capital, $\hat{K}_0$, and consumption in the present is obtained by borrowing against next period's output, $\hat{K}_1$.

The relevant Financial Market Line for the case $r > \rho^*$ (the dashed line) is sloped more steeply than the line for the best investment alternative, and therefore lies above it everywhere except at $\hat{K}_0$, the common point. Again, with the usual indifference map the preferred point will lie on this Financial Market Line, which implies some lending in the current period, as well as some consumption out of the capital stock $\hat{K}_0$.[32]

For example, an investor-consumer whose best investment alternative (with constant returns to scale) and capital $\hat{K}_0$ and indifference map as portrayed by the curve in the figure will act, depending on the market interest rate, as follows. With $r < \rho^*$ the solid Financial Market Line $K_0^*\hat{K}_1$ applies. The individual will invest his total capital, $\hat{K}_0$, in the productive investment and borrow OB' against next period's output, $\hat{K}_1$, allowing him to consume at point B: B' in the current period and B'' in the next period. If the market interest rate exceeds the rate of return on the best investment alternative, i.e., if $r > \rho^*$ (in which case the dashed Financial Market Line applies), this same individual will not invest in any productive investment in his own firm but will, instead, lend an amount $\hat{K}_0A'$ and consume A' in the current period and A'' in the next.

For both situations, consumption decisions are based on the market interest rate: the marginal time preference of the owner is equal to the interest rate. Since the optimal scale of investment is either zero or the maximum achievable as determined by a comparison of the rate of return with the interest rate, the Separation Theorem may be said to hold: the investment decision is independent of the tastes and preferences of the owner.

An explicit borrowing limit on the owner-firm with investments exhibiting constant returns to scale may be analyzed by means of Figure 8. Here only the best investment alternative is portrayed, and the amount of investment required for the maximum scale is assumed to be at least as large as the initial capital plus the maximum amount which may be borrowed. The line $\hat{K}_0\hat{K}_1$ which depicts the best investment alternative is projected into the second quadrant to point A, which represents the point of production employing all available resources: initial capital plus maximal borrowing. Assuming the rate of return on this investment alternative, ρ^*, to be higher than the market interest rate (or the borrowing rate, if there is a divergence between borrowing and lending rates), the "Consumption Possibilities Line" generated by this alternative is parallel to $\hat{K}_0\hat{K}_1$ but above it. Its construction

31. I.e., infinite.

32. The end points of the Financial Market Line may be ruled out since they imply zero consumption in one of the two periods.

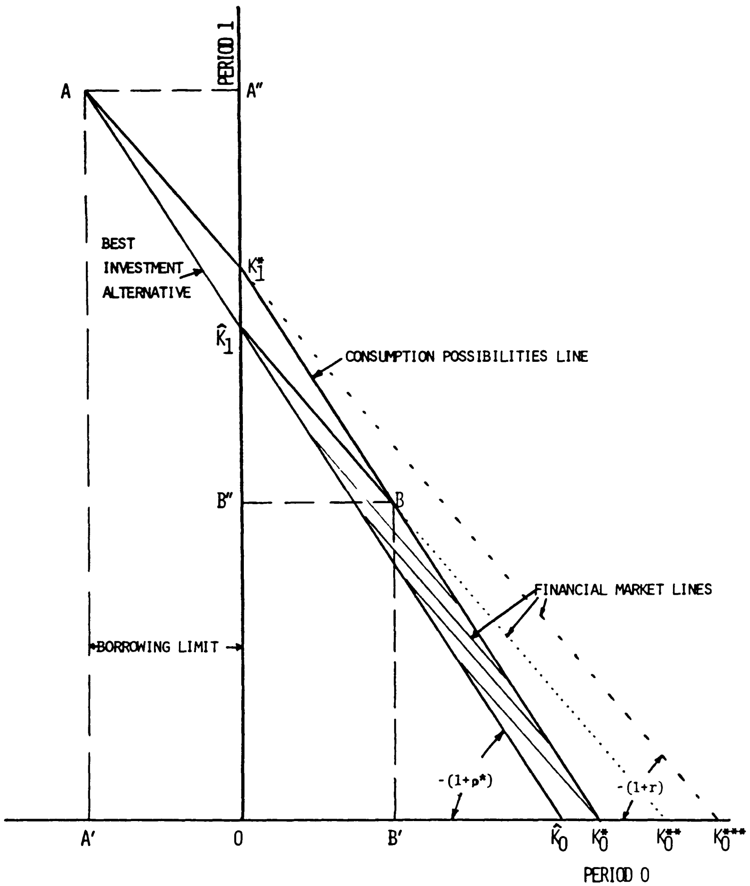

FIGURE 8. CAPITAL RATIONING AND CONSTANT RETURNS TO SCALE

is analogous to that of Figure 2. From each point on the Productive Opportunities Line, which here is the line representing the best investment alternative, a Financial Market Line segment is drawn such that its end point represents the net amount available for consumption in each of the two periods given both investment and borrowing. Considering point A, for example, the amount of investment is given by the distance $A'\hat{K}_0$, and the resulting output next period by $AA'=A''0$. The amount borrowed, assumed to be exactly equal to the borrowing limit, is $A'0$ requiring repayment $A''K_1^* = (1+r)A'$. The amount left the investor is then K_1^* in Period 1, but 0 in period 0. Alternatively, if only $\hat{K}_0$ is invested in the current period, the payoff next period will be $\hat{K}_1$, and by borrowing the maximum amount, the investor is able to consume at point B:[33] B' in the current period and B'' in the next. Because of the borrowing limit, the portion of the Financial Market Line below B (i.e., along $K_0^{**}B$) is unavailable to the owner of this investment opportunity. If up to $\hat{K}_0$ is invested in the current period combined with borrowing the

33. $A'0=0B'$ by construction.

maximum amount, investors may consume at any point along the line $K_0^* K_1^*$, including points below B.

When the investor's indifference map has the usual shape, the optimal consumption point will be the point of tangency between an indifference curve and the line $K_0^* K_1^*$, which here is called the "Consumption Possibilities Line" since it graphs the points of maximum consumption in one period for given amounts of consumption in the other. As the slope of this line is determined by the rate of return on the best investment alternative, ρ^* (assumed to be greater than the interest rate), the optimal consumption point is therefore one at which the marginal time preference of the owner is equal to this rate, and not to the market rate of interest, r. In addition, the firm cannot decide the optimal amount of investment without reference to the tastes and preferences of the owner. Therefore the Separation Theorem fails: investment and consumption decisions are mutually dependent on each other.[34] In sum, with constant returns to scale and limitations in the amount of borrowing, the firm should select that single investment with the highest average and marginal rates of return (and also here the highest internal rate of return). This information is not sufficient for the firm, unfortunately, since the *amount* of investment will still depend on the tastes and preferences of the owners.[35]

Constant returns to scale with limited borrowing by the firm but unlimited borrowing by its owners alters these conclusions. Assuming that the owners are prevented from circumventing the borrowing limit which the firm faces, e.g., by lending to the firm, we must still distinguish between the case in which the corporate borrowing rate is the same, and that in which it is lower than the personal borrowing rate. In the former case it would not benefit the owners if the firm were to make any positive amount of investment so long as the rate of return on the best investment alternative is less than the interest rate, i.e., $\rho^* < r$. If $\rho^* > r$, the owners would want the firm to invest the maximum amount possible in the best alternative, the initial amount, $\hat{K}_0$, plus the maximum the firm may borrow. The owners would then choose their preferred consumption vector based on their tastes and on the market interest rate.[36] Once more the Separation Theorem holds. Further, the firm will have made its investment decision by following a rule to

34. Multiple owners with distinct indifference maps present other difficulties. Since the optimal amount of investment depends on the shape of the indifference map of each of the owners, the set of "Pareto Optimal" amounts of investment with borrowing limits proportional to ownership may be dominated by different amounts of investment and borrowing totalling the same limited amount, but disproportionately distributed among the owners, e.g., by lending and borrowing among the set of owners at the market rate of interest. The implied incentive for purely financial transactions between the owners suggests that the investment decision process becomes more complex if also control of the firm is shared among a number of owners. This proposition also applies when there are non-constant returns to scale on investment.

35. The case $\rho^* < r$ may be ruled out on the following grounds. While in a formal sense the owner here would choose to lend at r rather than to invest in productive assets, unless the amount $\hat{K}_0$ represents liquid assets, their market value would fall so that the assets would return $\rho^* = r$, and the owner would be indifferent between making productive or financial investments.

36. The Consumption Possibilities Line for a single owner with no other assets here would be line $K_0^{***} K_1^*$.

maximize the terminal value of investment, or alternatively the discounted value of investment at the market interest rate, both subject to the constraint on corporate borrowing. Reference to the owners' indifference map is not required.[37] This conclusion requires only slight modification if the personal borrowing rate exceeds the corporate rate so long as the personal rate is not higher than ρ^*, the rate of return on the best investment alternative. By maximizing terminal value of investment (borrowing the maximum amount and achieving point K_1^* in Figure 7) investors would base their consumption decisions on this value and the personal borrowing rate.[38]

For a single owner, a personal borrowing rate greater than ρ^* (while the corporate rate is less than ρ^*) makes personal borrowing unattractive. Instead, the firm invests a smaller than maximum amount in its best alternative, borrowing the maximum on corporate account, while the owner does not borrow on private account. As a result, the scale of investment once more depends on the indifference map of the owner, and the previously discussed difficulties arise when there are multiple owners. It should be emphasized that these conclusions apply only when the personal borrowing rate is greater than the rate of return on the best investment alternative, and when at the same time, the corporate borrowing rate is less.[39]

VII. Linear Utility Functions

Having observed the difficulties with the criterion problem under strict capital rationing, Baumol and Quandt, as mentioned, turned to an explicit utility-of-consumption formulation to resolve the impasse.[40] For "simplicity" they chose a utility function which is linear in consumption in each period, 'spurning an esoteric appearance of greater profundity' [1, n. 2, p. 326] by employing a non-linear utility function. An unstated advantage they gained is that such a model would apply even for firms whose single owner derives income also from other sources, because linear utility functions imply constant marginal utility of money (consumption) and constant ratios of marginal utility of consumption between periods, and therefore, additivity.

In showing that the price paid for this simplification was too high, the present

37. Note that here no discount rate may be determined which serves as a substitute for the borrowing constraint, as was the case with a convex Productive Opportunity Line portrayed in Figure 3. This is precisely the issue dealt with in [17, Sec. 4.3].

38. The relevant personal financial market line for a single owner has not been drawn in Figure 8. It would extend from point K_1^* down to a point on the horizontal axis between K_0^* and K_0^{***}, its exact location depending on the personal borrowing rate.

39. A personal lending rate equal to the personal borrowing rate (or simply greater than ρ^*) would alter these conclusions. In this situation the firm would borrow the maximum amount but would undertake no productive investment in excess of this quantity. Investors would then plan their consumption from present value K_0^* by lending various amounts, depending on their tastes and preferences.

40. Hirshleifer, using a two-period analysis, found an indifference map sufficient for his analysis. This author in earlier work avoided specifying a utility function because of the difficulties arising with multiple owners having possibly different utility function. See [8] and [17: n. 1, p. 158; n. 31, p. 174].

analysis will also retain their assumption of constant returns to scale. Baumol and Quandt [1] made both assumptions and many authors[41] followed them in this. Once again, the tool of analysis will be the two-period Fisherian framework, as portrayed in Figure 9. This figure is identical to Figure 8 with only the indifference "curves" drawn in. With the owner-firm limited in the amount of borrowing, the "Consumption Possibilities Line" is again $K_0^*K_1^*$. The indifference curves are now parallel straight lines whose slope is the ratio of the (constant) marginal utility of consumption in period 0 to marginal utility of consumption in period 1, $-U_0/U_1$, with the negative sign denoting the tradeoff.

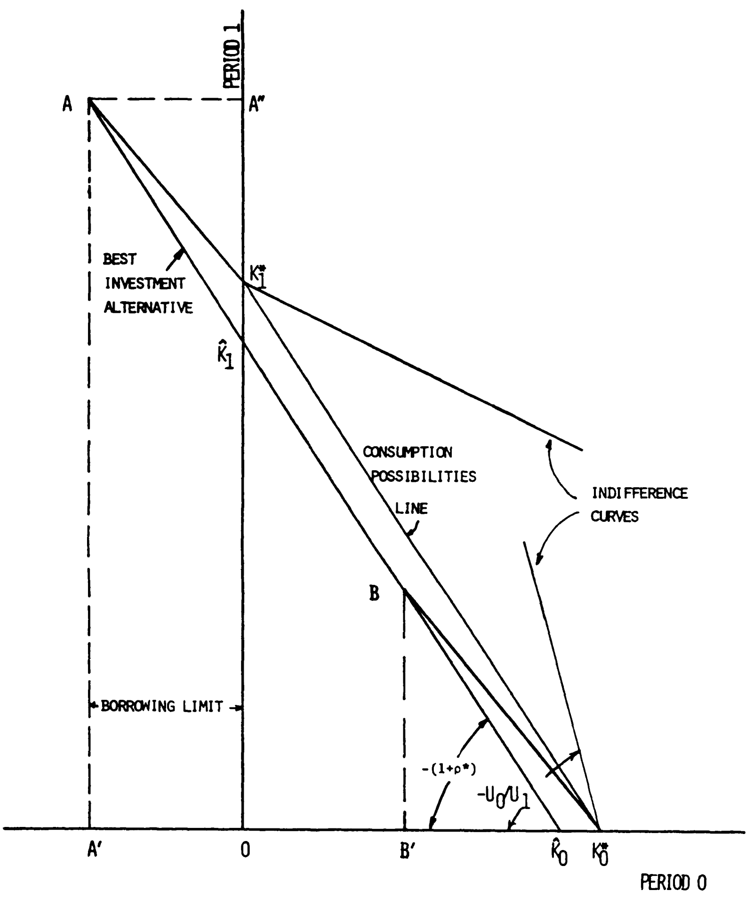

FIGURE 9. CAPITAL RATIONING: CONSTANT RETURNS TO SCALE AND LINEAR UTILITY FUNCTIONS

Three possible cases need to be examined: U_0/U_1 greater than, equal to, or less than $1+\rho^*$. If the ratio of utilities is greater than the return on the best investment alternative plus unity (i.e., if the indifference curve is steeper than the

41. See, for example, [2, 3, 15, 16] but specifically excepting Manne [14].

"Consumption Possibilities Line"), then the utility maximizing investor will choose to borrow the maximum permissible amount and invest an amount equal to $\hat{K}_0 - B'$. From the proceeds at B he will repay the loan, enabling him to consume K_0^* in the current period, but leaving nothing for the next period. Alternatively, if the ratio of utilities is less than $1+\rho^*$, this investor will defer all consumption until the next period and invest the maximum amount and borrow up to the limit. If, per chance, his indifference curves have the same slope as the Consumption Possibilities Line, which is the same as the slope of the best investment alternative, then and only then may the optimal consumption vector imply positive amounts of consumption in both periods! One may expect this coincidence to be rare, and hence the consequences of assuming linear utility of consumption—consumption in one period or in the other but not in both—makes this assumption unacceptable for the analysis of capital investment under capital rationing.[42]

VIII. Quest for the Elusive Discount Rate (s)

Among the early respondents to the Baumol and Quandt challenge to improve upon their subjective utility approach came Lusztig and Schwab [13]. In searching for means to alleviate "severe limitations" to which the application of linear programming models of capital budgeting were still subject [p. 427] they sought to focus on the "problem of the mutual dependence between the optimal solution of the linear programming model and the discount rate used to calculate the coefficients of its objective function."[43] Apparently they were after a method to find a single discount rate for the Lorie-Savage-Weingartner problem (with constant returns to scale) to compute new present values of the projects. This rate should have the property that the re-computed present values of accepted investments would exhaust the funds available from outside the firm plus funds generated by other accepted investments.

Passing over any question of their success, for the moment, one is forced to ask what purpose a solution to their quest would serve. For the two-period case without constant returns to scale, the answer to their question is provided geometrically in Figure 3 by the slope of line DF which determines the rate r', as was discussed earlier. Unfortuately, this rate may be found only after the decision problem has been solved, and therefore it is of no help in the formulation of a decision rule for capital budgeting when firms and owners are rationed, with which their paper presumably deals.[44]

As shown in the preceding section, when constant returns to scale are assumed (as Lusztig and Schwab have done), with a binding borrowing constraint the discount rate having the desired property is the marginal (and average) productiv-

42. Perfect capital markets and linear utility functions render this same conclusion, which also applies when the transformation function or Productive Opportunity Line is non-linear and convex.

43. Ibid.

44. For the constant returns to scale case which Lusztig and Schwab treated there does not necessarily exist a single rate, and with different rates for different periods the expenditure ceilings will still be necessary.

ity of the most productive investment, assuming the borrowing rate to be less than this. If it is higher, no borrowing takes place. With linear utility functions (i.e., indifference curves which are straight lines with negative slopes), the sought-after rate can only be bracketed, although the other difficulties pointed out earlier make the exercise pointless.[45]

More recently, Whitmore and Amey [20] sought to extend the mathematical results of Lusztig and Schwab, but under altered assumptions. The latter's statement of the problem was unclear to readers familiar with recent theory in corporate finance. It defined the cost of capital as the internal rate of return of the portfolio of foregone investments, which, though they did not appear to realize it, with constant returns to scale is the internal rate of return of the number of units of the investments in the optimal program. Whitmore and Amey, instead, allowed the discount rate to be different for different time periods, and interpreted the Lusztig and Schwab objective as "...essentially to obtain a price [i.e., discount] vector for which none of the budget constraints is binding" [20, pp. 128–29]. Elsewhere, however, they interpret both the work of Lusztig and Schwab as well as their own as being concerned with finding discount vectors which value investment cash flows at the rates implied by the constraints on expenditures, i.e., by the dual evaluators of the constraints. Clearly, these objectives are not the same. Burton and Damon [2] (as well as Baumol and Quandt) were also concerned with the latter formulation of the problem. Discussion of that second problem will be deferred for the moment.

Whitmore and Amey presented a more general model which included other constraints on investment, and allowed for nonconstant returns to scale. Although their language is not consistent on this point, they initially state that the expenditure celings are determined by the amount of funds generated from existing operations.[46] In constrast to borrowing limits imposed from outside, which represent not funds but potential funds which must be paid for, funds generated from operations have alternative uses. Whitmore and Amey therefore include discounted values of these funds in the objective function together with the discounted values of the project cash flows. This is equivalent to the maximization of the "horizon value" of the firm—investments plus cash generated from existing assets—as was done in the earliest of these models [17, Chapter 8]. Further, as discussed there [17, Sec. 9.1], the optimal values of the dual variables allow revaluation of the investments such that after revaluation the constraints become non-binding. While of interest for understanding the present value apparatus in capital budgeting, the

45. It is not clear from their text why they sought a single discount rate to apply to the cash flows in each period in recomputing present values, and they hardly provided proof that they had found it. For the purpose of this discussion it suffices to point out that the cost of capital is not necessarily constant through time. It is most commonly treated this way in corporate finance only because of the difficulties which pertain to this concept and its empirical estimation. Non-horizontal yield curves abound, and they suffice to indicate the ultimate need to generalize the cost of capital concept to a vector, rather than a scalar.

46. This was done in [17, Chapter 8] in the reformulation of the Lorie-Savage problem into a planning model.

revised discount rates are of no help in solving the capital investment decision problem because they are not available at the start.[47]

The interpretation of the problem which Lusztig and Schwab sought to solve as given above (and also found in one place in the Whitmore and Amey paper) relates to the following proposition. If, in a linear programming problem, an optimal solution imputes a set of shadow prices to the resources used to obtain that solution, then revising the coefficients of the objective function by appropriately increasing the cost of scarce resources will have a simple effect. The revised problem, with the constraints otherwise unchanged, can be made to have the same optimal solution in which none of the resource constraints is binding.[48] This follows directly from the dual theorem of linear programming.

While Whitmore and Amey[49] have generated such values, they have not solved the problem of consistency which, following Baumol and Quandt, they have stated. "The major difficulty...is the discrepancy that can arise between the price vector employed in the objective function and the true opportunity cost discount rates obtained from the optimal solution" [19, p. 128]. To solve this consistency problem implies, apparently, finding discount rates for investment cash flows which are *equal* to the optimal dual evaluators of the expenditure constraints. Although similarly seeking to find such quantitities, Burton and Damon [2] (who, unlike Whitmore and Amey, assumed constant returns to scale) ended by proving that the null discount vector—that thaving zero for all components—has the desired property. One must agree with their final paragraph, which begins, "...we can only conclude that this formulation of the pure capital rationing problem is meaningless. Thus we must search elsewhere for resolution"[50] [2, p. 1172]. The direction which this search might take is sketched out next.

47. Nevertheless, Whitmore and Amey found the exercise worthwhile, apparently, because they find consistent pricing "important" since "where a price mechanism is used to solve for the optimal investment program, the prices used to value cash flows in the objective function should be identical to the prices imputed to the budget constraints in the solution itself" [20, p. 134]. Unfortunately, they fail to mention what that price mechanism is and how it operates. (By contrasts, they regarded this author's use of market discount rates in [18, pp. 74–75] as "arbitrary" and unsatisfactory because the rates were not varied parametrically.)

48. Without presenting a mathematical statement of the issue, which is not essential here, it suffices to point out the following. Reducing the coefficients of the objective function by an amount equal to the sums of optimal dual values multiplied by the amount of resources used per unit of output will make the coefficients of the revised objective function either zero or negative. Variables that had positive optimal values will now have zero coefficients; variables at zero levels in the optimal solution will have negative coefficients. The original optimal solution will still be optimal, and the dual variables of the revised problem will be zero.

49. And if properly reinterpreted, also Lusztig and Schwab.

50. They arrive at the same conclusion with respect to Baumol and Quandt's utility of withdrawals formulation, in direct contradiction to that of Myers [16] who employed the exact same arguments, but by introducing an assertion about investor behavior, arrives at his conclusion about the role of market discount rates.

Actually, Burton and Damon's assertion that the null discount vector is unique turns out to be incorrect. Nevertheless, their conclusion with respect to the meaningfulness of such dual evaluators applies. Any other set of duals results in the same value of the primal and dual objective functions, namely zero.

IX. Conclusions

If nothing else, this review of the literature on capital rationing and the proper discount rates has shown that mere manipulation of mathematical models will not supply answers to substantive questions. Under different, though often unspecified sets of assumptions, different writers have often come to opposite conclusions, leaving readers in a quandary about the controversy. It bears reemphasizing that the original literature was concerned with managerial problems. Most subsequent writers have sought to interpret these models as solutions primarily to a problem in economics, without fully (or adequately) describing the market phenomena with which they were concerned. It may therefore be in order to comment more generally on capital rationing.

The form of capital rationing usually invoked in models just discussed involves an absolute limit on the funds available to a firm from outside—usually for an indefinite period. It is useful to categorize such situations into two kinds: those imposed from within, and those imposed by the capital market. Explicitly excluding from present discussion the process of capital budgeting within firms—i.e., setting expenditure ceilings for control purposes, which is not properly a case of capital rationing—one is left to deal with firms which choose to impose limits of their own volition, as well as those who, it is claimed, have them imposed on them.

Self-imposed expenditure limits, for working capital as well as for capital expenditures, may arise in small firms in which preservation of corporate control plays a dominant role. Thus, a small group of controlling owners may decide that the value of control is greater than any benefits likely to be derived by obtaining capital under conditions that imply actual or potential loss of control. They may prefer to consume less in order to retain the prerogatives of absolute control. Alternatively, they may realistically value the sale of a whole concern at a future date to be greater than the piecemeal sale that may permit faster growth.[51] Such a divergence between the market's valuation of the firm and that by controlling owners may be due to different assessments of the probabilities of success (i.e., different prior probabilities). Put more directly, when expenditure constraints are self-imposed, explanatory models must look beyond the maximization of the stream of consumption which investment in a firm may generate. Consideration of corporate control for closely held and professionally managed firms, as well as explicit recognition of uncertainty, must be incorporated in such models.[52]

Externally imposed capital rationing is a different issue. A number of writers[53] have cogently argued that the size and incidence of bankruptcy costs on lenders may make the supply curve for funds facing a given firm to become vertical. This comes about because, as evaluated by the capital markets, no amount of promised interest could compensate lenders for the risk of bankruptcy which the interest cost

51. Even allowing for discounting.

52. For example, a firm such as Eastman Kodak may be said to "deprive" its shareholders of the tax "subsidy" from debt financing, possibly because management finds it preferable not having banks or trustees under a bond indenture looking over their shoulders. The company's success in its main lines of business as well as its history of not borrowing may be averting shareholder pressure to alter this policy.

53. See, e.g., D. Hodgman [10] and subsequent *Comments* on this piece.

may itself produce. Since risk is the dominant consideration in this argument, clearly it should then be made an explicit part of theoretical models of capital expenditure decisions. However, few writers have been concerned with this, perhaps extreme, case.

There are periods of time when banks feel themselves to be "loaned up," i.e., under liquidity pressure such that they will not add new loan commitments and indeed may urge customers to hold down their borrowings under previously negotiated lines of credit. At such times, large firms may have access to funds by issuing their own commercial paper, but presumably at higher net cost since this alternative had also been available before a "credit squeeze." Whether, during tight money periods, banks allocate their loanable funds among all their borrowers or restrict lending to a chosen subset on the basis of total expected return to the bank is not really of consequence here. Insofar as it concerns a given firm, the supply schedule of funds may be positively sloped merely because substantial increases in desired funds requires tapping different sources. Higher information and operating costs, quite aside from risk differences that may be implied by substantial increases in borrowing, will increase the cost of obtaining funds from different kinds of institutions.

Periods of monetary tightness during which interest rates have risen relative to previously prevailing rates have not been infrequent in the post World War II period. During some of these periods stock prices have advanced while in others they declined, sometimes severely. Firms requiring funds for capital investment have sometimes found the equity market a good alternative to debt at times, but not at others. In other words, the cost of capital to business has not been constant, but has fluctuated and properly should be projected to fluctuate. Taking varying rates into account for capital budgeting purposes does not create substantial technical difficulties. The problems, which nevertheless are real, are those of estimating the appropriate rates to use and the organizational ones involved in their application.[54]

X. SUMMARY

The numerous published attempts to identify and treat the capital expenditure decisions problem reviewed here have considered capital rationing as taking place under naively and erroneously described capital markets. These efforts can only be regarded as massively counterproductive. Failure to identify just who is being rationed—firms and/or owners—and whether owners may successfully by-pass supposed market-imposed expenditure limits, as well as failure to consider whether the decision rules apply to individuals who own shares in several firms, or to firms owned by many owners has required this lengthy disentanglement of assumptions and conclusions found in the literature. The mathematical programming formulation of capital budgeting, although it has proved useful for analyzing and dealing with complex decision problems within firms, has been incorrectly and improperly

54. Indeed, if external capital rationing were expected to persist for long periods of time one would expect mergers between cash rich (or cash generating) firms and cash starved firms. Also, firms investing heavily in capital projects would not be expected to pay dividends at the same time.

applied in the literature to a problem that does not exist in anything resembling the form indicated by most writers. To lay this open to scrutiny this paper has examined the issues and divergent assumptions by adaptation of the simpler Fisherian framework, which permitted graphical portrayal of the analysis.[55]

To sum up, this paper has demonstrated the need for the continued development of capital budgeting under capital rationing as an aspect of the problem of decision and control within firms, and that the criterion as applied in the managerial literature does stand up under reasonable assumptions. Perhaps it will now be possible to retreat from sterile debates over decision rules and interpretations under naive capital rationing, and return to important problems of decision making under orgainzational constraints, as well as the separate study of the functioning of capital markets.

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55. In order not to leave the discount rate problem entirely without some resolution, with respect to the firm using self-imposed budgets (i.e., apart from this author's treatment in [18] it may be appropriate to make the following comments. First, a proper analysis would delve more deeply into the organizational and behavioral aspects surrounding the budgeting process: project generation, design, estimation, selection, implementation, control, and post-audit. Such an analysis has not been the objective here. Despite this, there is nothing in the present analysis which would bring into question the appropriateness of using the cost of capital as the discount rate when firms limit capital expenditures voluntarily. That doing so may impose losses on shareholders in the form of lost opportunities has been demonstrated above. Whether such opportunity losses are more than compensated for in the form of better planning and control is an issue which requires further study, and the attention of researchers is perhaps more profitably focussed in this direction.

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