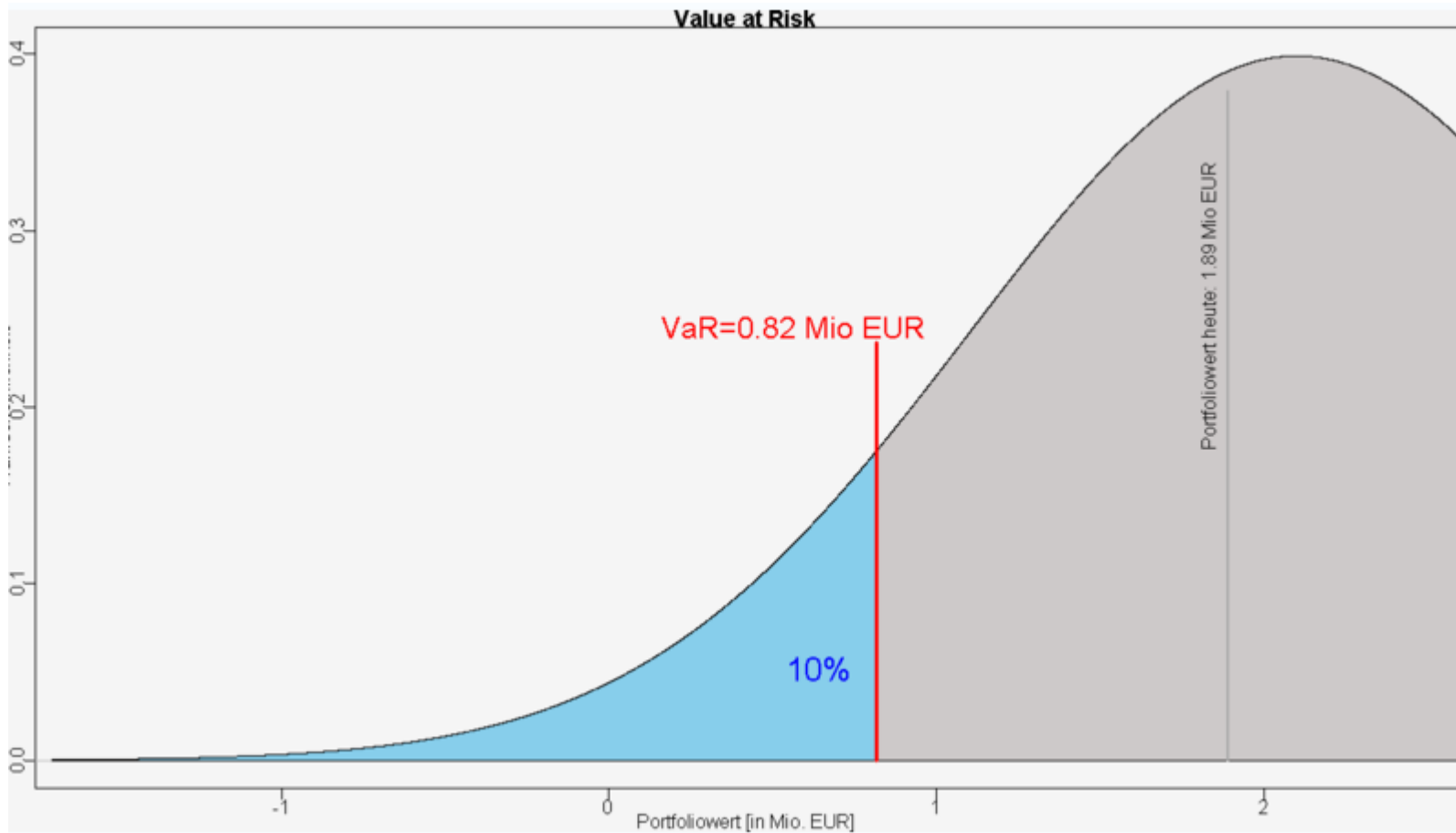


Value at Risk VaR

Value at Risk (VaR) is defined with respect to a specific portfolio of financial assets, at a specified probability and a specified time horizon. The probability that the mark-to-market loss on the portfolio over the time horizon is greater than VaR, assuming normal markets and no trading, is the specified probability level.

if a portfolio of stocks has a one-day 5% VaR of \$1 million, there is a 5% probability that the portfolio will decline in value by more than \$1 million over the next day, assuming markets are normal and there is no trading. Such an event is termed a “VaR break..

Value at Risk VaR



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Value at Risk VaR

- Common parameters for VaR are 1% and 5% probabilities and one day and two week horizons, although other combinations are in use.
- The reason for assuming normal markets and no trading, and to restricting loss to things measured in daily accounts, is to make the loss observable. In some extreme financial events it can be impossible to determine losses, either because market prices are unavailable or because the loss-bearing institution breaks up.
- Some longer-term consequences of disasters, such as lawsuits, loss of market confidence and employee morale and impairment of brand names can take a long time to play out, and may be hard to allocate among specific prior decisions. VaR marks the boundary between normal days and extreme events. Institutions can lose far more than the VaR amount, the only thing we can say is they won't do so very often.

Value at Risk VaR

Varieties of VaR

- The definition of VaR is nonconstructive, it specifies a property VaR must have, but not how to compute VaR. Moreover, there is wide scope for interpretation in the definition. This has led to two broad types of VaR, one used primarily in risk management and the other primarily for risk measurement. The distinction is not sharp, however and hybrid versions are typically used in financial control, financial reporting and computing regulatory capital.
- To a risk manager, VaR is a system, not a number. The system is run periodically (usually daily) and the published number is compared to the computed price movement in opening positions over the time horizon. There is never any subsequent adjustment to the published VaR, and there is no distinction between VaR breaks caused by input errors (including Information Technology breakdowns, fraud and rogue trading), computation errors (including failure to produce a VaR on time) and market movements.

Value at Risk VaR

- A frequentist claim is made, that the long-term frequency of VaR breaks will equal the specified probability, within the limits of sampling error, and that the VaR breaks will be independent in time and independent of the level of VaR.
- This claim is validated by a backtest, a comparison of published VaRs to actual price movements. In this interpretation, many different systems could produce VaRs with equally good backtests, but wide disagreements on daily VaR values.

Value at Risk VaR

- The term “VaR” is used both for a risk measure and a risk metric. This sometimes leads to confusion.
- Sources earlier than 1995 usually emphasize the risk measure, later sources are more likely to emphasize the metric.
- The VaR risk measure defines risk as mark-to-market loss on a fixed portfolio over a fixed time horizon, assuming normal markets. There are many alternative risk measures in finance. Instead of mark-to-market, which uses market prices to define loss, loss is often defined as change in fundamental value. For example, if an institution holds a loan that declines in market price because interest rates go up, but has no change in cash flows or credit quality, some systems do not recognize a loss.

Value at Risk VaR

- **Mark-to-market** is an accounting methodology of assigning a value to a position held in a financial instrument based on the current market price for the instrument or similar instruments. For example, the final value of a futures contract that expires in 9 months will not be known until it expires. If it is marked to market, for accounting purposes it is assigned the value that it would currently fetch in the open market.

Value at Risk VaR

- Or we could try to incorporate the economic cost of things not measured in daily financial statements, such as loss of market confidence or employee morale, impairment of brand names or lawsuits.
- Rather than assuming a fixed portfolio over a fixed time horizon, some risk measures incorporate the effect of expected trading (such as a stop loss order) and consider the expected holding period of positions. Finally, some risk measures adjust for the possible effects of abnormal markets, rather than excluding them from the computation.
- The VaR risk metric summarizes the distribution of possible losses by a quantile, a point with a specified probability of greater losses.

Value at Risk VaR

- Common alternative metrics are standard deviation, mean absolute deviation, expected shortfall and downside risk.
- Supporters of VaR-based risk management claim the first and possibly greatest benefit of VaR is the improvement in systems and modeling it forces on an institution.
- In 1997, Philippe Jorion wrote:
- The greatest benefit of VAR lies in the imposition of a structured methodology for critically thinking about risk. Institutions that go through the process of computing their VAR are forced to confront their exposure to financial risks and to set up a proper risk management function. Thus the process of getting to VAR may be as important as the number itself.

Value at Risk VaR

Publishing a daily number, on-time and with specified statistical properties holds every part of a trading organization to a high objective standard. Robust backup systems and default assumptions must be implemented. Positions that are reported, modelled or priced incorrectly stand out, as do data feeds that are inaccurate or late and systems that are too-frequently down.

Anything that affects profit and loss that is left out of other reports will show up either in inflated VaR or excessive VaR breaks. “A risk-taking institution that *does not* compute VaR might escape disaster, but an institution that *cannot* compute VaR will not.”

Recent events suggest otherwise!!

Value at Risk VaR

- The VaR risk measure is a popular way to aggregate risk across an institution. Individual business units have risk measures such as duration for a fixed income portfolio or beta for an equity business. These cannot be combined in a meaningful way.
- It is also difficult to aggregate results available at different times, such as positions marked in different time zones, or a high frequency trading desk with a business holding relatively illiquid positions. But since every business contributes to profit and loss in an additive fashion, and many financial businesses mark-to-market daily, it is natural to define firm-wide risk using the distribution of possible losses at a fixed point in the future.

Value at Risk VaR

- Risk measurement VaR is sometimes called parametric VaR. This usage can be confusing, however, because it can be estimated either parametrically (for examples, variance-covariance VaR) or nonparametrically (for examples, historical simulation VaR or resampled VaR).
- The inverse usage makes more logical sense, because risk management VaR is fundamentally nonparametric, but it is seldom referred to as nonparametric VaR.

Value at Risk VaR

History of VaR

- The problem of risk measurement is an old one in statistics, economics and finance. Financial risk management has been a concern of regulators and financial executives for a long time as well. Retrospective analysis has found some VaR-like concepts in this history.
- VaR did not emerge as a distinct concept until the late 1980s. The triggering event was the stock market crash of 1987. This was the first major financial crisis in which a lot of academically-trained quants were in high enough positions to worry about firm-wide survival.

Value at Risk VaR

- The crash was so unlikely given standard statistical models, that it called the entire basis of quant finance into question.
- A reconsideration of history led some quants to decide there were recurring crises, about one or two per decade, that overwhelmed the statistical assumptions embedded in models used for trading, investment management and derivative pricing.

Value at Risk VaR

- VaR was developed as a systematic way to segregate extreme events, which are studied qualitatively over long-term history and broad market events, from everyday price movements, which are studied quantitatively using short-term data in specific markets.
- Abnormal markets and trading were excluded from the VaR estimate in order to make it observable. It is not always possible to define loss if, for example, markets are closed as after 9/11, or severely illiquid, as happened several times in 2008. Losses can also be hard to define if the risk-bearing institution fails or breaks up. A measure that depends on traders taking certain actions, and avoiding other actions, can lead to self reference.

Value at Risk VaR

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Value at Risk VaR

- The financial events of the early 1990s found many firms in trouble because the same underlying bet had been made at many places in the firm, in non-obvious ways. Since many trading desks already computed risk management VaR, and it was the only common risk measure that could be both defined for all businesses and aggregated without strong assumptions, it was the natural choice for reporting firmwide risk.
- J. P. Morgan CEO Dennis Weatherstone famously called for a “4:15 report” that combined all firm risk on one page, available within 15 minutes of the market close.

Value at Risk VaR

- Risk measurement VaR was developed for this purpose. Development was most extensive at J. P. Morgan, which published the methodology and gave free access to estimates of the necessary underlying parameters in 1994.
- This was the first time VaR had been exposed beyond a relatively small group of quants.
- Two years later, the methodology was spun off into an independent for-profit business now part of RiskMetrics Group.
- In 1997, the U.S. Securities and Exchange Commission ruled that public corporations must disclose quantitative information about their derivatives activity. Major banks and dealers chose to implement the rule by including VaR information in the notes to their financial statements.

Value at Risk VaR

- Worldwide adoption of the Basel II Accord, beginning in 1999 and nearing completion today, gave further impetus to the use of VaR.
- VaR is the preferred measure of market risk, and concepts similar to VaR are used in other parts of the accord.

Value at Risk VaR

Mathematics

"Given some confidence level $\alpha \in (0, 1)$ the VaR of the portfolio at the confidence level α is given by the smallest number l such that the probability that the loss Z exceeds l is not larger than $(1 - \alpha)$ "^[3]

$$\text{VaR}_\alpha = \inf\{l \in \mathfrak{R} : P(L > l) \leq 1 - \alpha\} = \inf\{l \in \mathfrak{R} : F_L(l) \geq \alpha\}$$

- The left equality is a definition of VaR. The right equality assumes an underlying probability distribution, which makes it true only for parametric VaR.
- Risk managers typically assume that some fraction of the bad events will have undefined losses, either because markets are closed or illiquid, or because the entity bearing the loss breaks apart or loses the ability to compute accounts.

Value at Risk VaR

- Therefore, they do not accept results based on the assumption of a well-defined probability distribution. Nassim Taleb has labelled this assumption, "charlatanism."
- On the other hand, many academics prefer to assume a well-defined distribution, albeit usually one with fat tails. This point has probably caused more contention among VaR theorists than any other.

Value at Risk VaR

- Taleb claimed VaR:
- Ignored 2,500 years of experience in favor of untested models built by non-traders
- Was charlatanism because it claimed to estimate the risks of rare events, which is impossible
- Gave false confidence
- Would be exploited by traders
- More recently David Einhorn and Aaron Brown debated VaR in Global Association of Risk Professionals Review
- (See handout)

Value at Risk VaR

- Einhorn compared VaR to “an airbag that works all the time, except when you have a car accident.”
- He further charged that VaR:
- Led to excessive risk-taking and leverage at financial institutions
- Focused on the manageable risks near the center of the distribution and ignored the tails
- Created an incentive to take “excessive but remote risks”
- Was “potentially catastrophic when its use creates a false sense of security among senior executives and watchdogs.”

Value at Risk VaR

- A common complaint among academics is that VaR is not subadditive. That means the VaR of a combined portfolio can be larger than the sum of the VaRs of its components.
- As portfolios or institutions get larger, specific risks change from low-probability/low-predictability/high-impact to statistically predictable losses of low individual impact. That means they move from the range of far outside VaR, to be insured, to near outside VaR, to be analyzed case-by-case, to inside VaR, to be treated statistically.

Value at Risk VaR

Even VaR supporters generally agree there are common abuses of VaR:

- Referring to VaR as a "worst-case" or "maximum tolerable" loss. In fact, you expect two or three losses per year that exceed one-day 1% VaR.
- Making VaR control or VaR reduction the central concern of risk management. It is far more important to worry about what happens when losses exceed VaR.
- Assuming plausible losses will be less than some multiple, often three, of VaR. The entire point of VaR is that losses can be extremely large, and sometimes impossible to define, once you get beyond the VaR point.

Value at Risk VaR

- To a risk manager, VaR is the level of losses at which you stop trying to guess what will happen next, and start preparing for anything.
- Reporting a VaR that has not passed a backtest. Regardless of how VaR is computed, it should have produced the correct number of breaks (within sampling error) in the past. A common specific violation of this is to report a VaR based on the unverified assumption that everything follows a multivariate Normal distribution.