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Capital Markets V

Financial Instruments

Financial instruments is a term used to denote any form of funding medium - mostly those used for borrowing in money markets, e. g. bills of exchange, bonds, etc

Categorization

Financial instruments can be categorised by form depending on whether they are **cash instruments** or **derivative instruments**:

- **Cash instruments** are financial instruments whose value is determined directly by markets. They can be divided into [securities](#), which are readily transferable, and other *cash* instruments such as [loans](#) and [deposits](#), where both borrower and lender have to agree on a transfer.
- **Derivative instruments** are financial instruments which derive their value from some other financial instrument or variable. They can be divided into [exchange-traded derivatives](#) and [over-the-counter \(OTC\) derivatives](#).

Alternatively they can be categorised by "asset class" depending on whether they are **equity** based (reflecting [ownership](#) of the issuing entity) or **debt** based (reflecting a loan the investor has made to the issuing entity). If it is debt, it can be further categorised into **short term** (less than one year), medium (up to 5 years), or **long term**.

Foreign Exchange instruments and transactions are neither debt nor equity based and belong in their own category.

Matrix Table

Combining the above methods for categorisation, the main instruments can be organized into a matrix as follows:

Asset Class	Instrument Type			
	Securities	Other cash	Exchange-traded derivatives	OTC derivatives
Debt (Long Term plus medium term) >1 year	Bonds	Loans	Bond futures Options on bond futures	Interest rate swaps Interest rate caps and floors Interest rate options Exotic instruments
Debt (Short Term) <=1 year	Bills, e.g. T-Bills Commercial paper	Deposits Certificates of deposit	Short term interest rate futures	Forward rate agreements
Equity	Stock	N/A	Stock options Equity futures	Stock options Exotic instruments
Foreign Exchange	N/A	Spot foreign exchange	Currency futures	Foreign exchange options Outright forwards Foreign exchange swaps Currency swaps

Some instruments defy categorisation into the above matrix, for example [repurchase agreements](#).

For security (collateral), the legal right given to a [creditor](#) by a [borrower](#), see [security interest](#)

A **security** is a [fungible, negotiable instrument](#) representing financial value. Securities are broadly categorized into [debt](#) and [equity](#) securities such as [bonds](#) and [common stocks](#), respectively. The company or other entity issuing the security is called the issuer. What specifically qualifies as a security is dependent on the regulatory structure in a country. For example private investment pools may have some features

of securities, but they may not be registered or regulated as such if they meet various restrictions.

Securities may be represented by a certificate or, more typically, by an electronic book entry interest. Certificates may be bearer, meaning they entitle the holder to rights under the security merely by holding the security, or registered, meaning they entitle the holder to rights only if he or she appears on a security register maintained by the issuer or an intermediary. They include shares of corporate [stock](#) or [mutual funds](#), [bonds](#) issued by corporations or governmental agencies, [stock options](#) or other options, limited partnership units, and various other formal investment instruments that are negotiable and fungible.

Securities may be classified according to the following categories:

- Issuer
- Currency of denomination
- Ownership rights
- Term to maturity
- Degree of liquidity
- Income payments
- Tax treatment

By Type of Issuer

Issuers of securities include commercial companies, government agencies, local authorities and international and [supranational](#) organizations (such as the [World Bank](#)). Debt securities issued by a government (called [government bonds](#) or [sovereign bonds](#)) generally carry a lower interest rate than [corporate debt](#) issued by commercial companies. Interests in an asset -- for example, the flow of royalty payments from intellectual property—may also be turned into securities. These repackaged securities resulting from a [securitization](#) are usually issued by a company established for the purpose of the repackaging—called a special purpose vehicle (SPV). See "Repackaging" below. SPVs are also used to issue other kinds of securities. SPVs can also be used to guarantee securities, such as [covered bonds](#).

New capital: Commercial enterprises have traditionally used securities as a means of raising new capital. Securities may be an attractive option relative to bank loans depending on their pricing and market demand for particular characteristics. Another disadvantage of bank loans as a source of financing is that the bank may seek a measure of protection against default by the borrower via extensive financial covenants. Through securities, capital is provided by investors who purchase the securities upon their initial issuance. In a similar way, governments may raise capital through the issuance of securities (see [government debt](#)).

Repackaging: In recent decades securities have been issued to repackage existing assets. In a traditional securitisation, a financial institution may wish to remove assets from its [balance sheet](#) in order to achieve regulatory capital efficiencies or to accelerate its receipt of cash flow from the original assets. Alternatively, an

intermediary may wish to make a profit by acquiring financial assets and repackaging them in a way which makes them more attractive to investors.

By Type of Holder

Investors in securities may be [retail](#), i.e. members of the public investing other than by way of business. The greatest part in terms of volume of investment is [wholesale](#), i.e. by financial institutions acting on their own account, or on behalf of clients. Important [institutional investors](#) include [investment banks](#), [insurance](#) companies, [pension funds](#) and other managed funds.

Investment: The traditional economic function of the purchase of securities is investment, with the view to receiving [income](#) and/or achieving [capital gain](#). Debt securities generally offer a higher rate of interest than bank deposits, and equities may offer the prospect of capital growth. [Equity investment](#) may also offer control of the business of the issuer. Debt holdings may also offer some measure of control to the investor if the company is a fledgling start-up or an old giant undergoing 'restructuring'. In these cases, if interest payments are missed, the creditors may take control of the company and liquidate it to recover some of their investment.

Collateral: The last decade has seen an enormous growth in the use of securities as collateral. Purchasing securities with borrowed money secured by other securities is called "[buying on margin](#)." Where A is owed a debt or other obligation by B, A may require B to deliver [property rights](#) in securities to A. These property rights enable A to satisfy its claims in the event that B becomes [insolvent](#). Collateral arrangements are divided into two broad categories, namely security interests and outright collateral transfers. Commonly, commercial banks, investment banks and government agencies are significant collateral takers.

Debt and Equity

Securities are traditionally divided into debt securities and equities.

Debt

Debt securities may be called [debentures](#), [bonds](#), [notes](#) or [commercial paper](#) depending on their maturity and certain other characteristics. The holder of a debt security is typically entitled to the payment of principal and interest, together with other contractual rights under the terms of the issue, such as the right to receive certain information. Debt securities are generally issued for a fixed term and redeemable by the issuer at the end of that term. Debt securities may be protected by collateral or may be unsecured, and, if they are unsecured, may be contractually "senior" to other unsecured debt meaning their holders would have a priority in a bankruptcy of the issuer. Debt that is not senior is "subordinated".

[Corporate bonds](#) represent the debt of commercial or industrial entities. Debentures have a long maturity, typically at least ten years, whereas notes have a shorter maturity. Commercial paper is a simple form of debt security that essentially represents a post-dated check with a maturity of not more than 270 days.

Money market instruments are short term debt instruments that may have characteristics of deposit accounts, such as [certificates of deposit](#), and certain [bills of exchange](#). They are highly liquid and are sometimes referred to as "near cash". Commercial paper is also often highly liquid.

Euro debt securities are securities issued internationally outside their domestic market in a denomination different from that of the issuer's domicile. They include eurobonds and euronotes. Eurobonds are characteristically underwritten, and not secured, and interest is paid gross. A euronote may take the form of euro-commercial paper (ECP) or euro-certificates of deposit.

Government bonds are medium or long term debt securities issued by sovereign governments or their agencies. Typically they carry a lower rate of interest than corporate bonds, and serve as a source of finance for governments. U.S. federal government bonds are called *treasuries*. Because of their liquidity and perceived low risk, treasuries are used to manage the money supply in the [open market operations](#) of non-US central banks.

Sub-sovereign government bonds, known in the U.S. as [municipal bonds](#), represent the debt of state, provincial, territorial, municipal or other governmental units other than sovereign governments.

Supranational bonds represent the debt of international organizations such as the [World Bank](#), the [International Monetary Fund](#), regional [multilateral development banks](#) and others.

Equity

An equity security is a share in the capital stock of a company (typically common stock, although preferred equity is also a form of capital stock). The holder of an equity is a shareholder, owning a share, or fractional part of the issuer. Unlike debt securities, which typically require regular payments (interest) to the holder, equity securities are not entitled to any payment. In bankruptcy, they share only in the residual interest of the issuer after all obligations have been paid out to creditors. However, equity generally entitles the holder to a pro rata portion of control of the company, meaning that a holder of a majority of the equity is usually entitled to control the issuer. Equity also enjoys the right to [profits](#) and [capital gain](#), whereas holders of debt securities receive only interest and repayment of [principal](#) regardless of how well the issuer performs financially. Furthermore, debt securities do not have voting rights outside of bankruptcy. In other words, equity holders are entitled to the "upside" of the business and to control the business.

- [Stock](#)

Hybrid

Hybrid securities combine some of the characteristics of both debt and equity securities.

Preference shares form an intermediate class of security between equities and debt. If the issuer is liquidated, they carry the right to receive interest and/or a return of capital in priority to ordinary shareholders. However, from a legal perspective, they are capital stock and therefore may entitle holders to some degree of control depending on whether they contain voting rights.

Convertibles are bonds or preferred stock which can be converted, at the election of the holder of the convertibles, into the common stock of the issuing company. The convertibility, however, may be forced if the convertible is a callable bond, and the issuer calls the bond. The bondholder has about 1 month to convert it, or the company will call the bond by giving the holder the call price, which may be less than the value of the converted stock. This is referred to as a forced conversion.

Equity warrants are options issued by the company that allows the holder of the warrant to purchase a specific number of shares at a specified price within a specified time. They are often issued together with bonds or existing equities, and are, sometimes, detachable from them and separately tradable. When the holder of the warrant exercises it, he pays the money directly to the company, and the company issues new shares to holder.

Warrants, like other convertible securities, increases the number of shares outstanding, and are always accounted for in financial reports as fully diluted earnings per share, which assumes that all warrants and convertibles will be exercised.

The Securities Market

Primary and Secondary Market

The public securities markets can be divided into primary and secondary markets. The distinguishing difference between the two markets is that in the primary market, the money for the securities is received by the issuer of those securities from investors, whereas in the secondary market, the money goes from one investor to the other. When a company issues public stock for the first time, this is called an [Initial Public Offering](#) (IPO). A company can later issue more new shares, or issue shares that have been previously registered in a shelf registration. These later new issues are also sold in the primary market, but they are not considered to be an IPO. Issuers usually retain investment banks to assist them in administering the IPO, getting SEC approval, and selling the new issue. When the investment bank buys the entire new issue from the issuer at a discount to resell it at a markup, it is called an [underwriting](#), or firm commitment. However, if the investment bank considers the risk too great for an underwriting, it may only assent to a best effort agreement, where the investment bank will simply do its best to sell the new issue.

In order for the primary market to thrive, there must be a secondary market, or [aftermarket](#), where holders of securities can sell them to other investors for cash, hopefully at a profit. Otherwise, few people would purchase primary issues, and, thus, companies and governments would be unable to raise money for their operations.

Organized exchanges constitute the main secondary markets. Many smaller issues and most debt securities trade in the decentralized, dealer-based [over-the-counter](#) markets.

In Europe, the principal trade organization for securities dealers is the [International Capital Market Association](#). In the U.S., the principal organization for securities dealers is the [Securities Industry and Financial Markets Association](#). The [Bond Market Association](#) represents bond dealers globally.

Public Offer and Private Placement

In the primary markets, securities may be offered to the public in a [public offer](#). Alternatively, they may be offered privately to a limited number of qualified persons in a [private placement](#). Often a combination of the two is used. The distinction between the two is important to securities regulation and [company law](#). Privately placed securities are often not publicly tradable and may only be bought and sold by sophisticated qualified investors. As a result, the secondary market is not as liquid.

Another category, sovereign debt, is generally sold by auction to a specialised class of dealers.

Listing and OTC Dealing

Securities are often listed in a [stock exchange](#), an organised and officially recognised market on which securities can be bought and sold. Issuers may seek listings for their securities in order to attract investors, by ensuring that there is a liquid and regulated market in which investors will be able to buy and sell securities.

Growth in informal electronic trading systems has challenged the traditional business of stock exchanges. Large volumes of securities are also bought and sold "over the counter" (OTC). OTC dealing involves buyers and sellers dealing with each other by telephone or electronically on the basis of prices that are displayed electronically, usually by commercial information vendors such as [Reuters](#) and [Bloomberg](#).

There are also eurosecurities, which are securities that are issued outside their domestic market into more than one jurisdiction. They are generally listed on the [Luxembourg Stock Exchange](#) or admitted to listing in [London](#). The reasons for listing eurobonds include regulatory and tax considerations, as well as the investment restrictions.

International Debt Market

London is the centre of the eurosecurities markets. There was a huge rise in the eurosecurities market in London in the early 1980s. Settlement of trades in eurosecurities is currently effected through two European computerised systems called [Euroclear](#) (in Belgium) and [Clearstream](#) (formerly Cedelbank in Luxembourg).

The main market for Eurobonds is the EuroMTS, owned by Borsa Italiana and Euronext.

Physical Nature of Securities

Certificated Securities

Securities that are represented by certificates are called certificated securities. They may be *bearer* or *registered*.

Bearer Securities

Bearer securities are completely negotiable and entitle the holder to the rights under the security (e.g. to payment if it is a debt security, and voting if it is an equity security). They are transferred by delivering the instrument from person to person. In some cases, transfer is by endorsement, or signing the back of the instrument, and delivery.

Regulatory and fiscal authorities sometimes regard bearer securities negatively, as they may be used to facilitate the evasion of regulatory restrictions and tax. In the [United Kingdom](#), for example, the issue of bearer securities was heavily restricted firstly by the *Exchange Control Act 1947* until *1963*. Bearer securities are very rare in the United States because of the negative tax implications they may have to the issuer and holder.

Registered Securities

In the case of registered securities, certificates bearing the name of the holder are issued, but these merely represent the securities. A person does not automatically acquire legal ownership by having possession of the certificate. Instead, the issuer (or its appointed agent) maintains a register in which details of the holder of the securities are entered and updated as appropriate. A transfer of registered securities is effected by amending the register.

Uncertificated Securities and Global Certificates

Modern practice has developed to eliminate both the need for certificates and maintenance of a complete security register by the issuer. There are two general ways this has been accomplished.

Uncertificated Securities

In some jurisdictions, such as France, it is possible for issuers of that jurisdiction to maintain a legal record of their securities electronically...

Global Certificates and Book Entry Interests

In the United States, the corporation laws typically do not permit securities to be issued without being represented by one or more registered certificates. In order to facilitate the electronic transfer of interests in securities, a system has developed whereby issuers deposit a single global certificate representing all the outstanding securities of a class or series with a universal depository. This depository is called the

[Depository Trust Corporation](#), or DTC. DTC is a non-profit cooperative owned by approximately thirty of the largest Wall Street players that typically act as brokers or dealers in securities. These thirty banks are called the DTC participants. DTC, through a legal nominee, owns each of the global securities on behalf of all the DTC participants.

All securities traded through DTC are in fact held, in electronic form, on the books of various intermediaries between the ultimate owner, e.g. a retail investor, and the DTC participants. For example, Mr. Smith may hold 100 shares of Coca Cola, Inc. in his brokerage account at local broker Jones & Co. brokers. In turn, Jones & Co. may hold 1000 shares of Coca Cola on behalf of Mr. Smith and nine other customers. These 1000 shares are held by Jones & Co. in an account with Goldman Sachs, a DTC participant, or in an account at another DTC participant. Goldman Sachs in turn may hold millions of Coca Cola shares on its books on behalf of hundreds of brokers similar to Jones & Co. Each day, the DTC participants settle their accounts with the other DTC participants and adjust the number of shares held on their books for the benefit of customers like Jones & Co. Ownership of securities in this fashion is called beneficial ownership. Each intermediary holds on behalf of someone beneath him in the chain. The ultimate owner is called the beneficial owner. This is also referred to as owning in "Street name".

Other Depositories: Euroclear and Clearstream

Besides DTC, two other large securities depositories exist, both in Europe: Euroclear and Clearstream.

Divided and Undivided Security

The terms "divided" and "undivided" relate to the [proprietary](#) nature of a security.

Each divided security constitutes a separate asset, which is legally distinct from each other security in the same issue. Pre-electronic bearer securities were divided. Each instrument constitutes the separate covenant of the issuer and is a separate debt.

With undivided securities, the entire issue makes up one single asset, with each of the securities being a fractional part of this undivided whole. Shares in the secondary markets are always undivided. The issuer owes only one set of obligations to shareholders under its memorandum, articles of association and company law. A [share](#) represents an undivided fractional part of the issuing company. Registered debt securities also have this undivided nature.

Fungible and Non-fungible Security

The terms "fungible" and "non-fungible" relate to the way in which securities are held.

If an asset is fungible, this means that when such an asset is lent, or placed with a custodian, it is customary for the borrower or custodian to be obliged at the end of the loan or custody arrangement to return assets equivalent to the original asset, rather

than the identical asset. In other words, the redelivery of fungibles is equivalent and not *in specie* (identical).

Undivided securities are always fungible by logical necessity. Divided securities may or may not be fungible, depending on market practice. The clear trend is towards fungible arrangements.

Regulation

In the United States, the public offer and sale of securities must be either registered pursuant to a registration statement that is filed with the [U.S. Securities and Exchange Commission](#) (SEC) or are offered and sold pursuant to an exemption therefrom. Dealing in securities is heavily regulated by both the federal authorities (SEC) and state authorities. In addition the industry is heavily self policed by Self Regulatory Organizations (SROs), such as the NASD or the MSRB.

Due to the difficulty of creating a general definition that covers all securities, Congress attempts to define "securities" exhaustively (and not very precisely) as: "any [note](#), [stock](#), [treasury stock](#), [security future](#), [bond](#), [debenture](#), [certificate of interest](#) or participation in any [profit-sharing agreement](#) or in any oil, gas, or other [mineral royalty](#) or [lease](#), any [collateral-trust certificate](#), [preorganization certificate](#) or subscription, [transferable share](#), [investment contract](#), [voting-trust certificate](#), [certificate of deposit](#) for a security, any [put](#), [call](#), [straddle](#), [option](#), or privilege on any security, [certificate of deposit](#), or group or [index of securities](#) (including any interest therein or based on the value thereof), or any [put](#), [call](#), [straddle](#), [option](#), or privilege entered into on a national [securities exchange](#) relating to [foreign currency](#), or in general, any instrument commonly known as a 'security'; or any certificate of interest or participation in, temporary or interim certificate for, receipt for, or warrant or right to subscribe to or purchase, any of the foregoing; but shall not include [currency](#) or any [note](#), [draft](#), [bill of exchange](#), or [bankers' acceptance](#) which has a [maturity](#) at the time of issuance of not exceeding *nine months*, exclusive of days of grace, or any renewal thereof the maturity of which is likewise limited." - Section 3a item 10 of the 1934 Act.

The US Courts have developed a broad definition for securities that must then be registered with the SEC. There is an investment of money, a common enterprise and expectation of profits to come primarily from the efforts of others. See [SEC v. W.J. Howey Co.](#) and [SEC v. Glenn W. Turner Enterprises, Inc.](#)

In [finance](#), a **bond** is a [debt security](#), in which the authorized issuer owes the holders a debt and is obliged to repay the principal and interest (the [coupon](#)) at a later date, termed maturity. Other stipulations may also be attached to the bond issue, such as the obligation for the issuer to provide certain information to the bond holder, or limitations on the behavior of the issuer. Bonds are generally issued for a fixed term (the [maturity](#)) longer than ten years. U.S Treasury securities issue debt with life of ten years or more, which is a bond. New debt between one year and ten years is a "note", and new debt less than a year is a "bill".

A bond is simply a [loan](#), but in the form of a security, although terminology used is rather different. The *issuer* is equivalent to the *borrower*, the *bond holder* to the *lender*, and the *coupon* to the *interest*. Bonds enable the issuer to finance long-term [investments](#) with external funds. [Certificates of deposit](#) (CDs) or [commercial paper](#) are considered [money market](#) instruments.

In some nations, both *bonds* and *notes* are used irrespective of the maturity. Market participants normally use *bonds* for large issues offered to a wide public, and *notes* for smaller issues originally sold to a limited number of investors. There are no clear demarcations. There are also "bills" which usually denote fixed income securities with three years or less, from the issue date, to maturity. Bonds have the highest risk, notes are the second highest risk, and bills have the least risk. This is due to a statistical measure called [duration](#), where lower durations have less risk, and are associated with shorter term obligations.

Bonds and [stocks](#) are both [securities](#), but the difference is that stock holders own a part of the issuing company (have an [equity](#) stake), whereas bond holders are in essence lenders to the issuer. Also bonds usually have a defined term, or maturity, after which the bond is redeemed whereas stocks may be outstanding indefinitely. An exception is a [consol](#) bond, which is a [perpetuity](#) (i.e. bond with no maturity).

Features of bonds

The most important features of a bond are:

- **nominal, principal or face amount**—the amount over which the issuer pays interest, and which has to be repaid at the end.
- **issue price**—the price at which investors buy the bonds when they are first issued, typically \$1,000.00. The net proceeds that the issuer receives are calculated as the issue price, less issuance fees, times the nominal amount.
- **maturity date**—the date on which the issuer has to repay the nominal amount. As long as all payments have been made, the issuer has no more obligations to the bond holders after the maturity date. The length of time until the maturity date is often referred to as the **term** or **maturity** of a bond. The maturity can be any length of time, although debt securities with a term of less than one year are generally designated money market instruments rather than bonds. Most bonds have a term of up to thirty years. Some bonds have been issued with maturities of up to one hundred years, and some even do not mature at all. In early [2005](#), a market developed in [euros](#) for bonds with a maturity of fifty years. In the market for U.S. Treasury securities, there are three groups of bond maturities:
 - short term (bills): maturities up to one year;
 - medium term (notes): maturities between one and ten years;
 - long term (bonds): maturities greater than ten years.
- **coupon**—the interest rate that the issuer pays to the bond holders. Usually this rate is fixed throughout the life of the bond. It can also vary with a [money](#)

[market index](#), such as [LIBOR](#), or it can be even more exotic. The name coupon originates from the fact that in the past, physical bonds were issued which had coupons attached to them. On coupon dates the bond holder would give the coupon to a bank in exchange for the interest payment.

- **coupon dates**—the dates on which the issuer pays the coupon to the bond holders. In the U.S., most bonds are **semi-annual**, which means that they pay a coupon every six months. In Europe, most bonds are **annual** and pay only one coupon a year.
- **indenture** or **covenants**—a document specifying the rights of bond holders. In the U.S., federal and state securities and commercial laws apply to the enforcement of those documents, which are construed by courts as contracts. The terms may be changed only with great difficulty while the bonds are outstanding, with amendments to the governing document generally requiring approval by a majority (or super-majority) vote of the bond holders.
- **Optionality**: a bond may contain an **embedded option**; that is, it grants [option like](#) features to the buyer or issuer:
 - **callability**—Some bonds give the issuer the right to repay the bond before the maturity date on the [call dates](#); see [call option](#). These bonds are referred to as [callable bonds](#). Most callable bonds allow the issuer to repay the bond at [par](#). With some bonds, the issuer has to pay a premium, the so called [call premium](#). This is mainly the case for high-yield bonds. These have very strict covenants, restricting the issuer in its operations. To be free from these covenants, the issuer can repay the bonds early, but only at a high cost.
 - **puttability**—Some bonds give the bond holder the right to force the issuer to repay the bond before the maturity date on the put dates; see [put option](#).
 - **call dates** and **put dates**—the [dates](#) on which callable and puttable bonds can be redeemed early. There are four main categories.
 - A **Bermudan callable** has several call dates, usually coinciding with coupon dates.
 - A **European callable** has only one call date. This is a special case of a Bermudan callable.
 - An **American callable** can be called at any time until the maturity date.
 - A **death put** is an optional redemption feature on a debt instrument allowing the beneficiary of the estate of the deceased to put (sell) the bond (back to the issuer) in the event of the beneficiary's death or legal incapacitation. Also known as a "survivor's option".
- **sinking fund** provision of the corporate bond indenture requires a certain portion of the issue to be retired periodically. The entire bond issue can be liquidated by the maturity date. If that is not the case, then the remainder is called **balloon maturity**. Issuers may either pay to trustees, which in turn call randomly selected bonds in the issue, or, alternatively, purchase bonds in open market, then return them to trustees.

- [convertible bond](#) lets a bondholder exchange a bond to a number of shares of the issuer's common stock.
- [exchangeable bond](#) allows for exchange to shares of a corporation other than the issuer.

Types of bond

- [Fixed rate bonds](#) have a coupon that remains constant throughout the life of the bond.
- [Floating rate notes](#) (FRN's) have a coupon that is linked to a [money market index](#), such as [LIBOR](#) or [Euribor](#), for example three months USD LIBOR + 0.20%. The coupon is then reset periodically, normally every three months.
- [High yield bonds](#) are bonds that are rated below investment grade by the [credit rating agencies](#). As these bonds are relatively risky, investors expect to earn a higher yield. These bonds are also called [junk bonds](#).
- [Zero coupon bonds](#) do not pay any interest. They trade at a substantial discount from [par value](#). The bond holder receives the full principal amount as well as value that has accrued on the redemption date. An example of zero coupon bonds are Series E savings bonds issued by the U.S. government. [Zero coupon bonds](#) may be created from fixed rate bonds by financial institutions by "stripping off" the coupons. In other words, the coupons are separated from the final principal payment of the bond and traded independently.
- [Inflation linked bonds](#), in which the principal amount is indexed to inflation. The interest rate is lower than for fixed rate bonds with a comparable maturity. However, as the principal amount grows, the payments increase with inflation. The [government of the United Kingdom](#) was the first to issue inflation linked [Gilts](#) in the 1980s. [Treasury Inflation-Protected Securities](#) (TIPS) and [I-bonds](#) are examples of inflation linked bonds issued by the U.S. government.
- Other **indexed bonds**, for example [equity linked notes](#) and bonds indexed on a business indicator (income, added value) or on a country's [GDP](#).
- [Asset-backed securities](#) are bonds whose interest and principal payments are backed by underlying cash flows from other assets. Examples of asset-backed securities are [mortgage-backed securities](#) (MBS's), [collateralized mortgage obligations](#) (CMOs) and [collateralized debt obligations](#) (CDOs).
- [Subordinated bonds](#) are those that have a lower priority than other bonds of the issuer in case of [liquidation](#). In case of bankruptcy, there is a hierarchy of creditors. First the [liquidator](#) is paid, then government taxes, etc. The first bond holders in line to be paid are those holding what is called senior bonds. After they have been paid, the subordinated bond holders are paid. As a result, the risk is higher. Therefore, subordinated bonds usually have a lower credit rating than senior bonds. The main examples of subordinated bonds can be

found in bonds issued by banks, and asset-backed securities. The latter are often issued in [tranches](#). The senior tranches get paid back first, the subordinated tranches later.

- **[Perpetual bonds](#)** are also often called [perpetuities](#). They have no maturity date. The most famous of these are the UK Consols, which are also known as Treasury Annuities or Undated Treasuries. Some of these were issued back in [1888](#) and still trade today. Some ultra long-term bonds (sometimes a bond can last centuries: West Shore Railroad issued a bond which matures in 2361 (i.e. 24th century)) are sometimes viewed as perpetuities from a financial point of view, with the current value of principal near zero.
- **[Bearer bond](#)** is an official certificate issued without a named holder. In other words, the person who has the paper certificate can claim the value of the bond. Often they are registered by a number to prevent counterfeiting, but may be traded like cash. Bearer bonds are very risky because they can be lost or stolen. Especially after federal income tax began in the United States, bearer bonds were seen as an opportunity to conceal income or assets.^[1] U.S. corporations stopped issuing bearer bonds in the 1960s, the U.S. Treasury stopped in 1982, and state and local tax-exempt bearer bonds were prohibited in 1983.^[2]
- **[Registered bond](#)** is a bond whose ownership (and any subsequent purchaser) is recorded by the issuer, or by a transfer agent. It is the alternative to a [Bearer bond](#). Interest payments, and the principal upon maturity, are sent to the registered owner.
- **[Municipal bond](#)** is a bond issued by a state, U.S. Territory, city, local government, or their agencies. Interest income received by holders of municipal bonds is often [exempt](#) from the federal [income tax](#) and from the income tax of the state in which they are issued, although municipal bonds issued for certain purposes may not be tax exempt.
- **[Book-entry bond](#)** is a bond that does not have a paper certificate. As physically processing paper bonds and interest coupons became more expensive, issuers (and banks that used to collect coupon interest for depositors) have tried to discourage their use. Some book-entry bond issues do not offer the option of a paper certificate, even to investors who prefer them.^[3]
- **[Lottery bond](#)** is a bond issued by a state, usually a European state. Interest is paid like a traditional fixed rate bond, but the issuer will redeem randomly selected individual bonds within the issue according to a schedule. Some of these redemptions will be for a higher value than the face value of the bond.
- **[War bond](#)** is a bond issued by a country to fund a war.

Bonds issued by foreign entities

Some companies, banks, governments, and other sovereign entities may decide to issue bonds in foreign currencies as it may appear to be more stable and predictable

than their domestic currency. Issuing bonds denominated in foreign currencies also gives issuers the ability to access investment capital available in foreign markets. The proceeds from the issuance of these bonds can be used by companies to break into foreign markets, or can be converted into the issuing company's local currency to be used on existing operations. Foreign issuer bonds can also be used to hedge foreign exchange rate risk. Some of these bonds are called by their nicknames, such as the "samurai bond".

- **Eurodollar bond**, a U.S. dollar-denominated bond issued by a non-U.S. entity outside the U.S. [\[citation needed\]](#)
- **Kangaroo bond**, an Australian dollar-denominated bond issued by a non-Australian entity in the Australian market
- **Maple bond**, a Canadian Dollar-denominated bond issued by a non-Canadian entity in the Canadian market
- **Samurai bond**, a Japanese Yen-denominated bond issued by a non-Japanese entity in the Japanese market
- **Yankee bond**, a US Dollar-denominated bond issued by a non-US entity in the US market
- **Shogun bond**, a non-yen-denominated bond issued in Japan by a non-Japanese institution or government
- **Bulldog bond**, a pound sterling-denominated bond issued in London by a foreign institution or government
- **Ninja loan**, a Japanese yen syndicated loan by a foreign borrower [\[1\]](#)
- **Formosa bond**, a non-New Taiwan Dollar-denominated bond issued by a non-Taiwan entity in the Taiwan market [\[4\]](#)
- **Panda bond**, a Chinese renminbi-denominated bond issued by a non-China entity in the People's Republic of China market [\[5\]](#)

Current yield The present value relationship

The fair price of a straight bond (a bond with no [embedded option](#); see [Callable bond](#)) is determined by discounting the expected cash flows:

- Cash flows:
 - the periodic coupon payments **C**, each of which is made once every period;
 - the par or face value **F**, which is payable at maturity of the bond after **T** periods. (NB final year payment will include the par value plus the coupon payment for the year)
- Discount rate: the required (annually compounded) yield or rate of return **r**.
 - **r** is the market interest rate for new bond issues with similar risk ratings

$$\text{Bond Price} = P_0 = \sum_{t=1}^T \frac{C}{(1+r)^t} + \frac{F}{(1+r)^T}$$

Because the price is the present value of the cash flows, there is an inverse relationship between price and discount rate: the higher the discount rate the lower the

value of the bond (and vice versa). A bond trading below its face value is *trading at a discount*, a bond trading above its face value is *at a premium*.

Coupon yield

The [coupon yield](#) is simply the coupon payment (C) as a percentage of the face value (F).

$$\text{Coupon yield} = C / F$$

Coupon

The [current yield](#) is simply the coupon payment (C) as a percentage of the bond price (P).

$$\text{Current yield} = C / P_0.$$

Yield to Maturity

The [yield to maturity](#) (YTM) is the discount rate which returns the [market price](#) of the bond. It is thus the [internal rate of return](#) of an investment in the bond made at the observed price. YTM can also be used to price a bond, where it is used as the required return on the bond.

Solve for YTM where

$$\text{Market Price} = \sum_{t=1}^T \frac{C}{(1 + YTM)^t} + \frac{F}{(1 + YTM)^T}.$$

To achieve a return equal to YTM, the bond owner must:

- reinvest each coupon received at this rate,
- hold the bond until maturity, and
- redeem the bond at par.

The concept of current yield is closely related to other bond concepts, including yield to maturity, and coupon yield. The relationship between yield to maturity and coupon rate is as follows:

- When a bond sells at a discount, $YTM > \text{current yield} > \text{coupon yield}$.
- When a bond sells at a premium, $\text{coupon yield} > \text{current yield} > YTM$.
- When a bond sells at par, $YTM = \text{current yield} = \text{coupon yield}$.

The YTM is of limited use in valuing bonds with uncertain cash flows, such as [mortgage-backed securities](#) or [asset-backed securities](#). In these instances, other measures such as [option adjusted spread](#) should be used instead when comparing yields across different types of bonds.

Bond pricing

Relative price approach

Here the bond will be priced relative to a benchmark, usually a government security. The discount rate used to value the bond is determined based on the bond's rating relative to a government security with similar maturity or [duration](#). The better the quality of the bond, the smaller the spread between its required return and the YTM of the benchmark. This required return is then used to discount the bond cash flows as above.

Arbitrage free pricing approach

In this approach, the bond price will reflect its [arbitrage](#) free price (arbitrage=practice of taking advantage of a state of imbalance between two or more markets). Here, each cash flow is priced separately and is discounted at the same rate as the corresponding government issue [Zero coupon bond](#). (Some multiple of the bond (or the security) will produce an identical cash flow to the government security (or the bond in question).) Since each bond cash flow is known with certainty, the bond price today must be equal to the sum of each of its cash flows discounted at the corresponding [risk free rate](#) - i.e. the corresponding government security. Were this not the case, arbitrage would be possible - see [rational pricing](#).

Here the discount rate per cash flow, r_t , must match that of the corresponding zero coupon bond's rate. :

Bond Price =

$$P_0 = \sum_{t=1}^T \frac{C}{(1 + r_t)^t} + \frac{F}{(1 + r_T)^T}$$

Bond Duration

In [economics](#) and [finance](#), **duration** is the weighted average maturity of a [bond](#)'s cash flows or of any series of linked cash flows. Then the **duration** of a zero coupon bond with a maturity period of n years is n years. In case there will be coupon payments, the duration will be less than n years. This measure is closely related to the derivative of the bond's price function with respect to the interest rate, and some authors consider the duration to be this derivative, with the weighted average maturity simply being an easy method of calculating the duration for a non-callable bond. It is sometimes explained in inaccurate terms as being a measurement of how long, in years, it takes for the price of a bond to be **repaid** by its internal **cash flows**.

Price

Duration is useful as a measure of the sensitivity of a bond's price to [interest rate](#) movements. It is approximately inversely proportional to the percentage change in price for a given change in yield. For example, for small interest rate changes, the duration is the approximate percentage that the value of the bond will lose for a 1%

increase in interest rates. So a 15 year bond with a duration of 7 years would fall approximately 7% in value if the interest rate increased by 1%.

Basics

The standard definition of duration:

$$D = \sum_{i=1}^n \frac{P(i)t(i)}{V}$$

Where $P(i)$ is the [present value](#) of coupon i , $t(i)$ is the future payment date, V is the bond Price and D is the duration.

Cash Flow

As stated at the beginning, the **duration** is the weighted average maturity time of a bond cash flow. For a zero-coupon the duration will be $\Delta T = T_f - T_0$, where T_f is the maturity date and T_0 is the starting date of the bond. If there are different cash flows C_i the duration of every cash flow is $\Delta T_i = T_i - T_0$. From the current market price of the bond V , one can calculate the yield to maturity of the bond r using the formula

$$V = \sum_i C_i e^{-r\Delta T_i}$$

In a standard duration calculation, the overall yield of the bond is used to discount each cash flow leading to this expression in which the sum of the weights is 1:

$$D = \sum_i \Delta T_i \frac{C_i e^{-r\Delta T_i}}{V}$$

The higher the coupon rate from a bond, the shorter the duration. Duration is always less than or equal to the life (maturity) of a coupon bond. Only a zero coupon bond (a bond with no coupons) will have duration equal to the maturity.

Duration indicates also how much the value V of the bond changes in relation to a small change of the rate of the bond. We see that

$$\frac{\partial V}{\partial r} = - \sum_i \Delta T_i C_i e^{-r\Delta T_i} = -D \cdot V$$

then for small variation ∂r of the rate of the bond we have

$$\frac{\partial V}{V} = -D \partial r + O(\partial r^2)$$

That means that the duration gives the negative of the relative variation of the value of a bond respect to a variation of the rate of the bond, forgetting the quadratic terms. The quadratic terms are taken in account in the [Convexity](#).

Dollar Duration and Applications to [VaR](#)

The **Dollar duration** is defined as the product of the Duration and the price (value). It gives then the variation of a bond value for a small variation of the interest rate. Dollar duration $D_{\$}$ is commonly used for [VaR](#) (Value-at-Risk) calculation. If $V = V(r)$ denotes the value of a security depending on the interest rate r , dollar duration can be defined as

$$D_{\$} := -\frac{\partial V}{\partial r}.$$

To illustrate applications to portfolio risk management, consider a portfolio of securities dependent on the interest rates r_1, \dots, r_n as risk factors, and let $V = V(r_1, \dots, r_n)$

denote the value of such portfolio. Then the exposure vector $\omega = (\omega_1, \dots, \omega_n)$ has components

$$\omega_i = -D_{\$,i} := \frac{\partial V}{\partial r_i}$$

Accordingly, the change in value of the portfolio can be approximated as

$$\Delta V = \sum_{i=1}^n \omega_i \Delta r_i + \sum_{1 \leq i, j \leq n} O(\Delta r_i \Delta r_j)$$

that is, a component that is linear in the interest rate changes plus an error term which is at least quadratic. This formula can be used to calculate the [VaR](#) of the portfolio by ignoring higher order terms. Typically cubic or higher terms are truncated. Quadratic terms, when included, can be expressed in terms of (multi-variate) [bond convexity](#).

One can make assumptions about the joint distribution of the interest rates and then calculate [VaR](#) by Monte Carlo simulation or, in some special cases (e.g., Gaussian distribution assuming a linear approximation), even analytically. The formula can also be used to calculate the DV01 of the portfolio (cf. below) and it can be generalized to include risk factors beyond interest rates.

Macaulay duration

Macaulay duration, named for [Frederick Macaulay](#) who introduced the concept, is the weighted average maturity of a bond where the weights are the relative discounted cash flows in each period.

$$\text{Macaulay duration} = \frac{\sum (\text{present value of cash flow} \times \text{time to cash flow})}{\text{price of the bond}}.$$

Macaulay showed that an unweighted average maturity is not useful in predicting interest rate risk. He gave two alternative measures that are useful. The theoretically correct Macaulay-Weil duration which uses zero-coupon bond prices as discount factors, and the more practical form (shown above) which uses the bond's [yield to](#)

[maturity](#) to calculate discount factors. With the use of computers, both forms may be calculated, but the *Macaulay duration* is still widely used.

In case of *continuously compounded* yield the *Macaulay duration* coincides with the opposite of the partial derivative of the price of the bond with respect to the yield --as shown above. In case of yearly compounded yield, the modified duration coincides with the latter.

Modified duration

In case of *yearly compounded* yield the relation $\frac{\delta V}{V} = -D\delta r + O(\delta r^2)$ is not valid anymore. That is why the **modified duration** D^* is used instead:

$$D^* = \frac{\text{Macaulay duration}}{1 + \frac{r}{n}}$$

where r is the [yield to maturity](#) of the bond, and n is the number of cashflows per year.

Let us prove that the relation

$$\frac{\delta V}{V} = -D^*\delta r + O(\delta r^2)$$

is valid. We will analyze the particular case $n = 1$. The value (price) of the bond is

$$V = \sum_i \frac{C_i}{(1+r)^i}$$

where i is the number of years after the starting date the cash flow C_i will be paid. The duration, defined as the weighted average maturity, is then

$$D = \frac{1}{V} \sum_i \frac{C_i}{(1+r)^i} \cdot i$$

The derivative of V with respect to r is:

$$\frac{\partial V}{\partial r} = - \sum_i \frac{C_i}{(1+r)^{i+1}} \cdot i$$

multiplying by $\frac{(1+r)}{V}$ we obtain

$$\frac{\partial V}{\partial r} \cdot \frac{1+r}{V} = -D$$

or

$$\frac{\partial V}{\partial r} = -V \cdot D^*$$

from which we can deduce the formula

$$\frac{\delta V}{V} = -D^* \delta r + O(\delta r^2)$$

which is valid for yearly compounded yield.

Embedded options and effective duration

For bonds that have embedded options, Macauley duration and modified duration will not correctly approximate the price move for a change in yield. Consider a bond with an embedded put option. As an example, a \$1,000 bond that can be redeemed by the holder at par at points before the bond's maturity. No matter how high interest rates become, the price of the bond will never go below \$1,000. This bond's price sensitivity to interest rate changes is different from a non-puttable bond with identical cashflows. Bonds that have embedded options should be analyzed using "effective duration." Effective duration is a discrete approximation of the slope of the bond's value as a function of the interest rate.

$$\text{Effective Duration} = \frac{V_{-\Delta y} - V_{+\Delta y}}{2(V_0)\Delta y}$$

where Δy is the amount that yield changes, and $V_{-\Delta y}$ and $V_{+\Delta y}$ are the values that the bond will take if the yield falls by y or rises by y , respectively.

Average duration

The sensitivity of a [portfolio](#) of bonds such as a bond [mutual fund](#) to changes in interest rates can also be important. The average duration of the bonds in the portfolio is often reported. The duration of a portfolio equals the weighted average maturity of all of the cash flows in the portfolio. If each bond has the same [yield to maturity](#), this equals the weighted average of the portfolio's bond's durations. Otherwise the weighted average of the bond's durations is just a good approximation, but it can still

be used to infer how the value of the portfolio would change in response to changes in interest rates.

Bond duration closed-form formula

$$Dur = \frac{C(1+ai)(1+i)^m - (1+i) - (m-1+a)i}{i^2(1+i)^{(m-1+a)}} + \frac{100(m-1+a)}{(1+i)^{(m-1+a)}}$$

C = coupon payment per period (half-year)

i = discount rate per period (half-year)

a = fraction of a period remaining until next coupon payment

m = number of coupon dates until maturity

Convexity

Duration is a linear measure of how the price of a bond changes in response to interest rate changes. As interest rates change, the price does not change linearly, but rather is a convex function of interest rates. Convexity is a measure of the curvature of how the price of a bond changes as the interest rate changes. Specifically, duration can be formulated as the first derivative of the price function of the bond with respect to the interest rate in question, and the convexity as the second derivative.

Convexity also gives an idea of the spread of future cashflows. (Just as the duration gives the discounted mean term, so convexity can be used to calculate the discounted standard deviation, say, of return.)

PV01

PV01 is the present value impact of 1 basis point move in an interest rate. It is often used as a price alternative to duration (a time measure).

DV01

DV01 (Dollar Value of 1 basis point) is the same as PV01.

Trading and valuing bonds

The interest rate that the issuer of a bond must pay is influenced by a variety of factors, such as current market interest rates, the length of the term and the credit worthiness of the issuer.

These factors are likely to change over time, so the market value of a bond can vary after it is issued. Because of these differences in market value, bonds are priced in terms of percentage of par value. Bonds are not necessarily issued at par (100% of face value, corresponding to a price of 100), but all bond prices converge to par when they reach maturity. At other times, prices can either rise (bond is priced at greater

than 100), which is called trading at a premium, or fall (bond is priced at less than 100), which is called trading at a discount. Most government bonds are denominated in units of \$1000, if in the [United States](#), or in units of £100, if in the [United Kingdom](#). Hence, a deep discount US bond, selling at a price of 75.26, indicates a selling price of \$752.60 per bond sold. (Often, bond prices are quoted in points and thirty-seconds of a point, rather than in decimal form.) Some short-term bonds, such as the U.S. T-Bill, are always issued at a discount, and pay par amount at maturity rather than paying coupons. This is called a discount bond.

The market price of a bond is the [present value](#) of all future interest and principal payments of the bond discounted at the bond's [yield](#), or [rate of return](#). The yield represents the current market interest rate for bonds with similar characteristics. The yield and price of a bond are inversely related so that when market interest rates rise, bond prices generally fall and vice versa.

The [market price](#) of a bond may include the [accrued interest](#) since the last coupon date. (Some bond markets include accrued interest in the trading price and others add it on explicitly after trading.) The price including accrued interest is known as the "flat" or "[dirty price](#)". (See also [Accrual bond](#).) The price excluding accrued interest is sometimes known as the [Clean price](#).

The interest rate adjusted for the current price of the bond is called the "current yield" or "earnings yield" (this is the nominal yield multiplied by the par value and divided by the price).

Taking into account the expected [capital gain](#) or loss (the difference between the current price and the [redemption value](#)) gives the "redemption yield": roughly the current [yield](#) plus the capital gain (negative for loss) per year until redemption.

The relationship between yield and maturity for otherwise identical bonds is called a [yield curve](#).

Bonds markets, unlike stock or share markets, often do not have a centralized exchange or trading system. Rather, in most developed [bond markets](#) such as the U.S., Japan and western Europe, bonds trade in decentralized, dealer-based [over-the-counter](#) markets. In such a market, [market liquidity](#) is provided by dealers and other market participants committing risk capital to trading activity. In the bond market, when an investor buys or sells a bond, the counterparty to the trade is almost always a bank or securities firm acting as a dealer. In some cases, when a dealer buys a bond from an investor, the dealer carries the bond "in inventory." The dealer's position is then subject to risks of price fluctuation. In other cases, the dealer immediately resells the bond to another investor.

Bond markets also differ from stock markets in that investors generally do not pay brokerage commissions to dealers with whom they buy or sell bonds. Rather, dealers earn revenue for trading with their investor customers by means of the spread, or difference, between the price at which the dealer buys a bond from one investor--the "bid" price--and the price at which he or she sells the same bond to another investor--the "ask" or "offer" price. The [bid/offer spread](#) represents the total transaction cost associated with transferring a bond from one investor to another.

Stock or equity

Types of stock

Common stock

Common stock, also referred to as **common** or **ordinary shares**, are, as the name implies, the most usual and commonly held form of **stock** in a [corporation](#). The other type of shares that the public can hold in a corporation is known as [preferred stock](#). Common stock that has been re-purchased by the corporation is known as [treasury stock](#) and is available for a variety of corporate uses.

Common stock typically has voting rights in corporate decision matters, though perhaps different rights from preferred stock. In order of priority in a [liquidation](#) of a corporation, the owners of common stock are near the last. Dividends paid to the stockholders must be paid to preferred shares before being paid to common stock shareholders. [\[1\]](#)

Preferred stock

[Preferred stock](#), sometimes called preferred shares, have priority over common stock in the distribution of dividends and assets.

Most **preferred** shares provide no voting rights in corporate decision matters. However, some preferred shares have special voting rights to approve certain extraordinary events (such as the issuance of new shares, or the approval of the acquisition of the company), or to elect directors. [\[2\]](#)

Dual class stock

[Dual class stock](#) is shares issued for a single company with varying classes indicating different rights on voting and dividend payments. Each kind of shares has its own class of shareholders entitling different rights.

Treasury stock

[Treasury stock](#) are shares that have been bought back from public. Treasury Stock is considered issued, but not outstanding.

Golden share

[Golden share](#) is a special share giving its holder a right to veto the Board's decisions. Usually, a government owns golden shares of important enterprises that were privatized. Golden shares are mostly used in European countries.

Stock derivatives

A stock [derivative](#) is any financial claim which has a value that is dependent on the price of the [underlying](#) stock. [Futures](#) and [options](#) are the main types of derivatives on stocks. The underlying security may be a [stock index](#) or an individual firm's stock, e.g. [single-stock futures](#).

Stock futures are contracts where the buyer, or [long](#), takes on the obligation to buy on the contract maturity date, and the seller, or [short](#) takes on the obligation to sell. [Stock index futures](#) are generally not delivered in the usual manner, but by [cash settlement](#).

A [stock option](#) is a class of [option](#). Specifically, a [call option](#) is the right (*not* obligation) to buy stock in the future at a fixed price and a [put option](#) is the right (*not* obligation) to sell stock in the future at a fixed price. Thus, the value of a stock option changes in reaction to the underlying stock of which it is a [derivative](#). The most popular method of valuing stock options is the [Black Scholes](#) model [3].

Futures vs. Forwards

While futures and [forward contracts](#) are both a contract to deliver a commodity on a future date at a prearranged price, they are different in several respects:

- Forwards only transact when purchased and on the settlement date. Futures, on the other hand, are rebalanced, or "marked-to-market", everyday to the daily spot price of a forward with the same agreed-upon delivery price and underlying asset.
 - The lack of rebalancing of forwards means that, in some cases, due to movements in the underlying's price, a large differential will build up between the forward's delivery price and the settlement price.
 - This means that one party will incur a big loss at the time of delivery (assuming they must transact at the underlying's spot price to facilitate receipt/delivery).
 - This in turn creates a credit risk. More generally, the risk of a forward contract is that the supplier will be unable to deliver the required commodity, or that the buyer will be unable to pay for it on the delivery day.
 - The rebalancing of futures eliminates much of this credit risk by forcing the holders to update daily to the price of an equivalent forward purchased that day. This means that there will usually be very little additional money due on the final day to settle the future.
 - In addition, the daily futures settlement failure risk is borne by an exchange, rather than an individual party, thus further reducing credit risk in futures.
 - Example for a future with a \$100 futures price: Let's say that on day 50, a forward with a \$100 delivery price (on the same underlying asset as the future) costs \$88. On day 51, that forward costs, say, \$90. This means that the mark-to-market would require the holder of one side of the future to pay \$2 on day 51 to track the changes of the forward price. This money goes, via margin accounts, to the holder of the other

side of the future. (A forward-holder, however, would pay nothing until settlement on the final day, potentially building-up a large balance. So, except for tiny effects of convexity bias or possible allowance for credit risk, futures and forwards with equal delivery prices result in the same total loss or gain, but holders of futures experience that loss/gain in daily increments which track the forward's daily price changes, while the forward's spot price converges to the settlement price.)

- Futures are always traded on an [exchange](#), whereas forwards always trade [over-the-counter](#), or can simply be a signed contract between two parties.
- Futures are highly standardised, whereas some forwards are unique.
- In the case of physical delivery, the forward contract specifies to whom to make the delivery. The counterparty for delivery on a futures contract is chosen by the [clearinghouse](#).

○

Standardization

Margin

Settlement

Pricing

The situation where the price of a commodity for future delivery is higher than the spot price, or where a far future delivery price is higher than a nearer future delivery, is known as [contango](#). The reverse, where the price of a commodity for future delivery is lower than the spot price, or where a far future delivery price is lower than a nearer future delivery, is known as [backwardation](#).

When the deliverable asset exists in plentiful supply, or may be freely created, then the price of a future is determined via [arbitrage](#) arguments. The forward price represents the expected future value of the underlying [discounted](#) at the [risk free rate](#)—as any deviation from the theoretical price will afford investors a riskless profit opportunity and should be arbitrated away; see [rational pricing of futures](#).

Thus, for a simple, non-dividend paying asset, the value of the future/forward, $F(t)$, will be found by compounding the present value $S(t)$ at time t to maturity T by the rate of risk-free return r .

$$F(t) = S(t) \times (1 + r)^{(T-t)}$$

or, with *continuous compounding*

$$F(t) = S(t)e^{r(T-t)}$$

This relationship may be modified for storage costs, dividends, dividend yields, and convenience yields.

In a perfect market the relationship between futures and spot prices depends only on the above variables; in practice there are various market imperfections (transaction costs, differential borrowing and lending rates, restrictions on short selling) that prevent complete arbitrage. Thus, the futures price in fact varies within arbitrage boundaries around the theoretical price.

The above relationship, therefore, is typical for stock index futures, treasury bond futures, and futures on physical commodities when they are in supply (e.g. on corn after the harvest). However, when the deliverable commodity is not in plentiful supply or when it does not yet exist, for example on wheat before the harvest or on [Eurodollar Futures](#) or [Federal Funds Rate](#) futures (in which the supposed underlying instrument is to be created upon the delivery date), the futures price cannot be fixed by arbitrage. In this scenario there is only one force setting the price, which is simple supply and demand for the future asset, as expressed by supply and demand for the futures contract.

In a deep and liquid market, this supply and demand would be expected to balance out at a price which represents an unbiased expectation of the future price of the actual asset and so be given by the simple relationship

$$F(t) = E_t \{S(T)\} .$$

In fact, this relationship will hold in a no-arbitrage setting when we take expectations with respect to the [risk-neutral probability](#). In other words: a futures price is [martingale](#) with respect to the risk-neutral probability.

With this pricing rule, a speculator is expected to break even when the futures market fairly prices the deliverable commodity.

Futures contracts and exchanges

There are many different kinds of futures contracts, reflecting the many different kinds of tradable assets of which they are [derivatives](#). There are futures markets in specific underlying [commodity markets](#), such as:

- [Foreign exchange market](#)
- [Money market](#)
- [Bond market](#)
- [Equity index market](#)
- [Soft Commodities market](#)

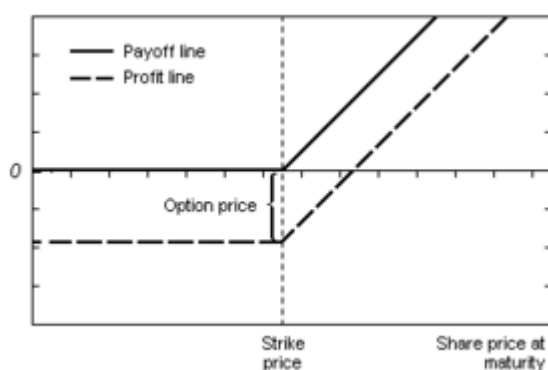
Trading on [commodities](#) began in Japan in the 18th century with the trading of rice and silk, and similarly in Holland with tulip bulbs. Trading in the US began in the mid 19th century, when central grain markets were established and a marketplace was created for farmers to bring their commodities and sell them either for immediate delivery (also called spot or cash market) or for forward delivery. These forward

contracts were private contracts between buyers and sellers and became the forerunner to today's exchange-traded futures contracts. Although contract trading began with traditional commodities such as grains, meat and livestock, exchange trading has expanded to include metals, energy, currency and currency indexes, equities and equity indexes, government interest rates and private interest rates.

Contracts on financial instruments was introduced in the 1970s by the [Chicago Mercantile Exchange](#)(CME) and these instruments became hugely successful and quickly overtook commodities futures in terms of trading volume and global accessibility to the markets. This innovation led to the introduction of many new futures exchanges worldwide, such as the [London International Financial Futures Exchange](#) in 1982 (now [Euronext.liffe](#)), Deutsche Terminbörse (now [Eurex](#)) and the [Tokyo Commodity Exchange](#) (TOCOM). Today, there are more than 75 futures and futures options exchanges worldwide trading to include:

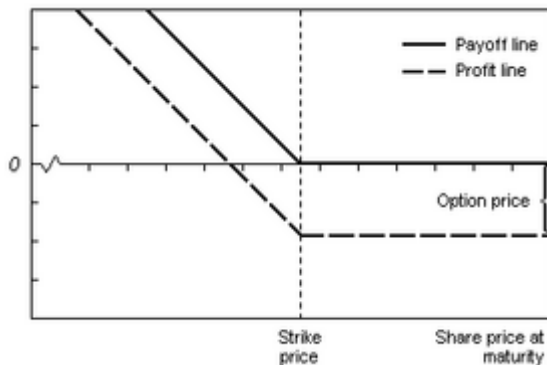
- [Chicago Board of Trade](#) (CBOT) -- Interest Rate derivatives (US Bonds); Agricultural (Corn, Soybeans, Soy Products, Wheat); Index (Dow Jones Industrial Average); Metals (Gold, Silver)
- [Chicago Mercantile Exchange](#) -- Currencies, Agricultural (Pork, Cattle, Butter, Milk); Index (NASDAQ, S&P, etc); Various Interest Rate Products
- ICE Futures - the International Petroleum Exchange trades energy including [crude oil](#), heating oil, [natural gas](#) and unleaded gas and merged with IntercontinentalExchange(ICE)to form ICE Futures.
- [Euronext.liffe](#)
- [Sydney Futures Exchange](#)
- London Commodity Exchange - softs: grains and meats. Inactive market in [Baltic Exchange](#) shipping.
- [Tokyo Commodity Exchange](#) TOCOM
- [London Metal Exchange](#) - metals: [copper](#), [aluminium](#), [lead](#), [zinc](#), [nickel](#) and [tin](#).
- [New York Board of Trade](#) - softs: [cocoa](#), [coffee](#), [cotton](#), [orange juice](#), [sugar](#)
- [New York Mercantile Exchange](#) - energy and metals: [crude oil](#), [gasoline](#), [heating oil](#), [natural gas](#), [coal](#), [propane](#), [gold](#), [silver](#), [platinum](#), [copper](#), [aluminum](#) and [palladium](#)
- [Futures exchange](#)
- OneChicago Futures on many [Single-stock futures](#)

Option (finance).





Payoffs and profits from a long call.



Payoffs and profits from a long put.

Options are financial instruments that convey the right, but not the obligation, to engage in a future transaction on some [underlying](#) security. For example, buying a [call option](#) provides the right to buy a specified amount of a security at a set [strike price](#) at some time on or before [expiration](#), while buying a [put option](#) provides the right to sell. Upon the option holder's choice to [exercise](#) the option, the party that sold, or wrote, the option must fulfill the terms of the contract.^[1]

The theoretical value of an option can be determined by a variety of techniques, including the use of sophisticated option valuation models. These models can also predict how the value of the option will change in the face of changing conditions. Hence, the risks associated with trading and owning options can be understood and managed with some degree of precision.

Exchange-traded options form an important class of options which have standardized contract features and trade on public exchanges, facilitating trading among independent parties. [Over-the-counter](#) options are traded between private parties, often well-capitalized institutions, that have negotiated separate trading and clearing arrangements with each other. Another important class of options, particularly in the U.S., are [employee stock options](#), which are awarded by a company to their employees as a form of incentive compensation.

Other types of options exist in many financial contracts, for example [real estate options](#) are often used to assemble large parcels of land, and [prepayment](#) options are usually included in [mortgage](#) loans. However, many of the valuation and risk management principles apply across all financial options.

Contract specifications

Every financial option is a contract between the two counterparties. Option contracts may be quite complicated, however, at minimum, they usually contain the following specifications:^[2]

- whether the option holder has the right to buy (a [call option](#)) or the right to sell (a [put option](#))

- the amount and class of the [underlying](#) asset(s) (e.g. 100 shares of XYZ Co. B stock)
- the [strike price](#), also known as the exercise price, which is the price at which the underlying transaction will occur upon [exercise](#)
- the [expiration](#) date, or expiry, which is the last date the option can be exercised
- the [settlement terms](#), for instance whether the writer must deliver the actual asset on exercise, or may simply tender the equivalent cash amount
- the terms by which the option is quoted in the market, usually a multiplier such as 100, to convert the quoted price into actual premium amount

Types of options

The primary types of financial options are:

- **Exchange traded options** (also called "listed options") is a class of [exchange traded derivatives](#). Exchange traded options have standardized contracts, and are settled through a [clearing house](#) with fulfillment guaranteed by the credit of the exchange. Since the contracts are standardized, accurate pricing models are often available. Exchange traded options include:^{[3][4]}
 1. stock options,
 2. commodity options,
 3. [bond options](#) and other [interest rate options](#)
 4. index (equity) options, and
 5. options on futures contracts
- **Over-the-counter, or OTC** options are traded between two private parties, and are not listed on an exchange. The terms of an OTC option are unrestricted and may be individually tailored to meet any business need. In general, at least one of the counterparties to an OTC option is a well-capitalized institution. Option types commonly traded over the counter include:
 1. interest rate options
 2. currency cross rate options, and
 3. options on [swaps](#) or [swaptions](#).
- **Employee stock options** are issued by a company to its employees as compensation.

Option Style

Naming conventions are used to help identify properties common to many different types of options. These include:

- **European** option - an option that may only be [exercised](#) on [expiration](#).
- **American** option - an option that may be exercised on any trading day on or before expiration.

- **Bermudan** option - an option that may be exercised only on specified dates on or before expiration.
- **Barrier** option - any option with the general characteristic that the underlying security's price must reach some trigger level before the exercise can occur.

Valuation models

The value of an option can be estimated using a variety of quantitative techniques, although most commonly through the use of option pricing models such as [Black-Scholes](#) and the [binomial options pricing model](#).^[5] In general, standard option valuation models depend on the following factors:

- The current market price of the underlying security,
- the [strike price](#) of the option, particularly in relation to the current market price of the underlier,
- the cost of holding a position in the underlying security, including interest and dividends,
- the time to [expiration](#) together with any restrictions on when exercise may occur, and
- an estimate of the future [volatility](#) of the underlying security's price over the life of the option.

More advanced models can require additional factors, such as an estimate of how volatility changes over time and for various underlying price levels, or the dynamics of stochastic interest rates.

The following are some of the principal valuation techniques used in practice to evaluate option contracts.

Black Scholes

The Black-Scholes model was the first quantitative technique to comprehensively and accurately estimate the price for a variety of simple option contracts. By employing the technique of constructing a risk neutral portfolio that replicates the returns of holding an option, [Fischer Black](#) and [Myron Scholes](#) produced a closed-form solution for a European option's theoretical price.^[6] At the same time, the model generates [hedge parameters](#) necessary for effective risk management of option holdings. While the ideas behind Black-Scholes were ground-breaking and eventually led to a [Nobel Prize in Economics](#) for [Myron Scholes](#) and [Robert Merton](#), application of the model in actual options trading is clumsy because of the assumptions of continuous (or no) dividend payment, constant volatility, and a constant interest rate. Nevertheless, the Black-Scholes model is still widely used in academic work, and for many financial applications where the model's error is within margin of tolerance.^[7]

The term **Black–Scholes** refers to three closely related concepts:

- The **Black–Scholes model** is a mathematical model of the market for an equity, in which the equity's price is a [stochastic process](#).

- The **Black–Scholes PDE** is an equation which (in the model) must be satisfied by the price of a derivative on the equity.
- The **Black–Scholes formula** is the result obtained by applying the Black-Scholes PDE to [European](#) put and call options.

[Robert C. Merton](#) was the first to publish a paper expanding our mathematical understanding of the [options](#) pricing model and coined the term "Black-Scholes" options pricing model, by enhancing work that was published by [Fischer Black](#) and [Myron Scholes](#). The paper was first published in [1973](#). The foundation for their research relied on work developed by scholars such as [Louis Bachelier](#), [A. James Boness](#), [Edward O. Thorp](#), and [Paul Samuelson](#). The fundamental insight of Black-Scholes is that the option is implicitly priced if the stock is traded.

Merton and Scholes received the 1997 [Nobel Prize in Economics](#) for this and related work; though ineligible for the prize because of his death in 1995, Black was mentioned as a contributor by the Swedish academy.

The model

The key assumptions of the Black–Scholes model are:

- The price of the [underlying instrument](#) S_t follows a [geometric Brownian motion](#) with [constant](#) drift μ and [volatility](#) σ :

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

- It is possible to [short sell](#) the underlying stock.
- There are no [arbitrage](#) opportunities.
- Trading in the stock is continuous.
- There are no [transaction costs](#) or [taxes](#).
- All securities are perfectly divisible (*e.g.* it is possible to buy 1/100th of a share).
- It is possible to borrow and lend cash at a constant [risk-free interest rate](#).
- The stock does not pay a dividend (see below for extensions to handle dividend payments).

The above lead to the following formula for the price of a [European call option](#) with exercise price K on a stock currently trading at price S , *i.e.*, the right to buy a share of the stock at price K after T years. The constant interest rate is r , and the constant stock [volatility](#) is σ .

$$C(S, T) = S\Phi(d_1) - Ke^{-rT}\Phi(d_2)$$

where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

Here Φ is the [standard normal cumulative distribution function](#).

The price of a [put option](#) may be computed from this by [put-call parity](#) and simplifies to

$$P(S, T) = Ke^{-rT}\Phi(-d_2) - S\Phi(-d_1).$$

[The Greeks](#) under the Black–Scholes model are calculated below:

	Calls	Puts
delta	$\Phi(d_1)$	$\Phi(d_1) - 1$
gamma	$\frac{\phi(d_1)}{S\sigma\sqrt{T}}$	
vega	$S\phi(d_1)\sqrt{T}$	
theta	$-\frac{S\phi(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}\Phi(d_2)$	$-\frac{S\phi(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}\Phi(-d_2)$
rho	$KT e^{-rT}\Phi(d_2)$	$-KT e^{-rT}\Phi(-d_2)$

Here, ϕ is the standard normal [probability density function](#). Note that the gamma and vega formulas are the same for calls and puts. This can be seen directly from [put-call parity](#).

In practice, some sensitivities are usually quoted in scaled-down terms, to match the scale of likely changes in the parameters. For example, rho is often reported divided by 10,000 (1bp rate change), vega by 100 (1 vol point change), and theta by 365 or 252 (1 day decay based on either calendar days or trading days per year).

Extensions of the model

The above model can easily be extended to have non-constant (but deterministic) rates and volatilities. The model may also be used to value European options on instruments paying dividends. In this case, closed-form solutions are available if the dividend is a known proportion of the stock price. [American options](#) and options on stocks paying a known cash dividend (in the short term, more realistic than a proportional dividend) are more difficult to value, and a choice of solution techniques is available (for example lattices and grids).

Instruments paying continuous yield dividends

For options on indexes (such as the [FTSE](#) where each of 100 constituent companies may pay a dividend twice a year and so there is a payment nearly every business day), it is reasonable to make the simplifying assumption that dividends are paid continuously, and that the dividend amount is proportional to the level of the index.

The dividend payment paid over the time period $[t, t + dt]$ is then modelled as

$$qS_t dt$$

for some constant q (the [dividend yield](#)).

Under this formulation the arbitrage-free price implied by the Black–Scholes model can be shown to be

$$C(S_0, T) = e^{-rT} (F\Phi(d_1) - K\Phi(d_2))$$

where now

$$F = S_0 e^{(r-q)T}$$

is the modified forward price that occurs in the terms d_1 and d_2 :

$$d_1 = \frac{\ln(F/K) + (\sigma^2/2)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}.$$

Exactly the same formula is used to price options on foreign exchange rates, except that now q plays the role of the foreign risk-free interest rate and S is the spot exchange rate. This is the **Garman–Kohlhagen model** (1983).

Instruments paying discrete proportional dividends

It is also possible to extend the Black–Scholes framework to options on instruments paying discrete proportional dividends. This is useful when the option is struck on a single stock.

A typical model is to assume that a proportion δ of the stock price is paid out at pre-determined times t_1, t_2, \dots . The price of the stock is then modelled as

$$S_t = S_0(1 - \delta)^{n(t)} e^{ut + \sigma W_t}$$

where $n(t)$ is the number of dividends that have been paid by time t .

The price of a call option on such a stock is again

$$C(S_0, T) = e^{-rT}(F\Phi(d_1) - K\Phi(d_2))$$

where now

$$F = S_0(1 - \delta)^{n(T)} e^{rT}$$

is the forward price for the dividend paying stock.

Black–Scholes in practice

The volatility smile

All the parameters in the model other than the volatility — the time to maturity, the [strike](#), the risk-free rate, and the current underlying price — are unequivocally observable. Furthermore, under normal circumstances the option's theoretical value is a monotonic increasing function of the volatility. This means there is a one-to-one relationship between the option price and the volatility. By computing the implied volatility for traded options with different strikes and maturities, we can test the Black-Scholes model. If the Black–Scholes model held, then the implied volatility for a particular stock would be the same for all strikes and maturities. In practice, the [volatility surface](#) (the three-dimensional graph of implied volatility against strike and maturity) is not flat. The typical shape of the implied volatility curve for a given maturity depends on the underlying instrument. Equities tend to have skewed curves: implied volatility is higher for low strikes, and slightly lower for high strikes. Currencies tend to have more symmetrical curves, with implied volatility lowest [at-the-money](#), and higher volatilities in both wings. Commodities often have the reverse behaviour to equities, with higher implied volatility for higher strikes.

Despite the existence of the volatility smile (and the violation of all the other assumptions of the Black-Scholes model), the Black-Scholes PDE and Black-Scholes formula are still used extensively in practice. A typical approach is to regard the volatility surface as a fact about the market, and use an implied volatility from it in a Black-Scholes valuation model. This has been described as using "the wrong number in the wrong formula to get the right price" [Rebonato 1999]. This approach also gives usable values for the hedge ratios (the Greeks).

Even when more advanced models are used, traders prefer to think in terms of volatility as it allows them to evaluate and compare options of different maturities, strikes, and so on.

Valuing bond options

Black–Scholes cannot be applied directly to [bond securities](#) because of the [pull-to-par](#) problem. As the bond reaches its maturity date, all of the prices involved with the bond become known, thereby decreasing its volatility, and the simple Black–Scholes model does not reflect this process. A large number of extensions to Black–Scholes, beginning with the [Black model](#), have been used to deal with this phenomenon.

Interest rate curve and short stock rate

One difficulty that often arises in practice is how to derive the proper interest rate to use as an input. The deposit rate for a risk-free bond maturing on the option's expiration date is, in general, not observable in the market. Instead, an [interest rate curve](#) is used. Composed of market-quoted interest rates of various maturities, the curve provides an estimate of the risk-free rate of appropriate maturity for the option being priced.

Another issue arises when [short stock](#) is to be used as part of the hedging portfolio. This is because your broker typically pays you some rate that is less than the risk-free rate on the proceeds of the short stock sale. In addition, when a stock is hard to borrow, the rate you receive on the short sale proceeds can go down and even be negative. That is, you might have to pay your broker interest on the proceeds from your short sale as an inducement to lend you the shares you have sold short. In these cases, the correct interest rate to use in the model should be adjusted to account for this effect.

For example, your broker pays you the Fed funds overnight rate less 0.85% (85 basis points) on your short stock proceeds. You have no existing position in IBM, but you are considering purchasing IBM Jan08 100 Calls. Because you would ordinarily sell IBM short to hedge this purchase, you will need to borrow IBM shares from your broker. From your interest rate curve, you determine the proper risk-free rate for a theoretical bond expiring on January 19th, 2008 is 5.05%. Therefore, the correct interest rate to use in the Black-Scholes model is 4.2%. Now, assume that you are considering the same trade, but in the symbol HLYS, which is hard to borrow. Your broker will only pay you 2% less than the overnight rate on proceeds from a short sale in HLYS stock. Now, the correct rate to use in the Black-Scholes model is 3.05%.

Formula derivation

Elementary derivation

Let S_0 be the current price of the underlying stock and S the price when the option matures at time T . Then S_0 is known, but S is a [random variable](#). Assume that

$$X \equiv \ln(S/S_0)$$

is a [normal random variable](#) with [mean](#) μT and [variance](#) $\sigma^2 T$. It follows that the mean of S is

$$\mathbb{E}[S] = S_0 e^{qT}$$

for some constant q (independent of T). Now a simple no-arbitrage argument shows that the theoretical future value of a derivative paying one share of the stock at time T , and so with payoff S , is

$$S_0 e^{rT}$$

where r is the risk-free interest rate. This suggests making the identification $q = r$ for the purpose of pricing derivatives. Define the theoretical value of a derivative as the [present value](#) of the [expected](#) payoff in this sense. For a call option with exercise price K this discounted expectation (using [risk-neutral probabilities](#)) is

$$C(S_0, T) = e^{-rT} \mathbb{E} [\max(S - K, 0)].$$

The derivation of the formula for C is facilitated by the following [lemma](#): Let Z be a [standard normal](#) random variable and let b be an [extended real number](#). Define

$$Z^+(b) = \begin{cases} Z & \text{if } Z > b \\ -\infty & \text{otherwise} \end{cases}.$$

If a is a positive real number, then

$$\mathbb{E} [e^{aZ^+(b)}] = e^{a^2/2} \Phi(-b + a)$$

where Φ is the standard normal [cumulative distribution function](#). In the special case $b = -\infty$, we have

$$\mathbb{E} [e^{aZ}] = e^{a^2/2}.$$

Now let

$$Z = \frac{X - uT}{\sigma\sqrt{T}}$$

and use the [corollary](#) to the lemma to verify the statement above about the mean of S . Define

$$S^+ = \begin{cases} S & \text{if } S > K \\ 0 & \text{otherwise} \end{cases}$$

$$X^+ = \ln(S^+/S_0)$$

and observe that

$$\frac{X^+ - uT}{\sigma\sqrt{T}} = Z^+(b)$$

for some b . Define

$$K^+ = \begin{cases} K & \text{if } S > K \\ 0 & \text{otherwise} \end{cases}$$

and observe that

$$\max(S - K, 0) = S^+ - K^+.$$

The rest of the calculation is straightforward.

Although the elementary derivation leads to the correct result, it is incomplete as it cannot explain, why the formula refers to the riskfree interest rate while a higher rate of return is expected from risky investments. This limitation can be overcome using the risk-neutral probability measure, but the concept of risk-neutrality and the related theory is far from elementary.

PDE based derivation

In this section we derive the [partial differential equation](#) (PDE) at the heart of the Black–Scholes model via a no-arbitrage or [delta-hedging](#) argument; for more on the underlying logic, see the discussion at [rational pricing](#).

The presentation given here is [informal](#) and we do not worry about the validity of moving between dt meaning a small increment in time and dt as a [derivative](#).

The Black–Scholes PDE

As per the model assumptions above, we assume that the [underlying](#) (typically the stock) follows a [geometric Brownian motion](#). That is,

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where W_t is Brownian.

Now let V be some sort of option on S —mathematically V is a function of S and t . $V(S, t)$ is the value of the option at time t if the price of the underlying stock at time t is S . The value of the option at the time that the option matures is known. To determine its value at an earlier time we need to know how the value evolves as we go backward in time. By [Itô's lemma](#) for two variables we have

$$dV = \left(\mu S \frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt + \sigma S \frac{\partial V}{\partial S} dW.$$

Now consider a trading strategy under which one holds one option and continuously trades in the stock in order to hold $-\partial V/\partial S$ shares. At time t , the value of these holdings will be

$$\Pi = V - S \frac{\partial V}{\partial S}.$$

The composition of this portfolio, called the [delta-hedge](#) portfolio, will vary from time-step to time-step. Let R denote the accumulated profit or loss from following this strategy. Then over the time period $[t, t + dt]$, the instantaneous profit or loss is

$$dR = dV - \frac{\partial V}{\partial S} dS.$$

By substituting in the equations above we get

$$dR = \left(\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt.$$

This equation contains no dW term. That is, it is entirely riskless ([delta neutral](#)). Thus, given that there is no arbitrage, the rate of return on this portfolio must be equal to the rate of return on any other riskless instrument. Now assuming the risk-free rate of return is r we must have over the time period $[t, t + dt]$

$$r\Pi dt = dR = \left(\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt.$$

If we now substitute in for Π and divide through by dt we obtain the **Black–Scholes PDE**:

$$\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0.$$

This is the law of evolution of the value of the option. With the assumptions of the Black–Scholes model, this equation holds whenever V has two derivatives with respect to S and one with respect to t .

Other derivations of the PDE

Above we used the method of [arbitrage](#)-free pricing ("[delta-hedging](#)") to derive a PDE governing option prices given the Black–Scholes model. It is also possible to use a [risk-neutrality](#) argument. This latter method gives the price as the [expectation](#) of the option payoff under a particular [probability measure](#), called the [risk-neutral measure](#), which differs from the real world measure.

Solution of the Black–Scholes PDE

We now show how to get from the general Black–Scholes PDE to a specific valuation for an option. Consider as an example the Black–Scholes price of a [call option](#) on a stock currently trading at price S . The option has an exercise price, or strike price, of K , i.e. the right to buy a share at price K , at T years in the future. The constant interest

rate is r and the constant stock volatility is σ . Now, for a call option the PDE above has [boundary conditions](#)

$$\begin{aligned} V(0, t) &= 0 \text{ for all } t \\ V(S, t) &\sim S \text{ as } S \rightarrow \infty \\ V(S, T) &= \max(S - K, 0). \end{aligned}$$

The last condition gives the value of the option at the time that the option matures. The solution of the PDE gives the value of the option at any earlier time. In order to solve the PDE we transform the equation into a [diffusion equation](#) which may be solved using standard methods. To this end we introduce the change-of-variable transformation

$$\begin{aligned} x &= \ln(S/K) + (r - \sigma^2/2)(T - t) \\ \tau &= T - t \\ u &= V e^{r(T-t)}. \end{aligned}$$

Then the Black–Scholes PDE becomes a diffusion equation

$$\frac{\partial u}{\partial \tau} = \frac{\sigma^2}{2} \frac{\partial^2 u}{\partial x^2}.$$

The terminal condition $V(S, T) = \max(S - K, 0)$ now becomes an initial condition

$$u(x, 0) = u_0(x) \equiv K \max(e^x - 1, 0).$$

Using the standard method for solving a diffusion equation we have

$$u(x, \tau) = \frac{1}{\sigma \sqrt{2\pi\tau}} \int_{-\infty}^{\infty} u_0(y) e^{-(x-y)^2/(2\sigma^2\tau)} dy.$$

After some algebra we obtain

$$u(x, \tau) = K e^{x+\sigma^2\tau/2} \Phi(d_1) - K \Phi(d_2)$$

where

$$\begin{aligned} d_1 &= \frac{x + \sigma^2\tau}{\sigma\sqrt{\tau}} \\ d_2 &= \frac{x}{\sigma\sqrt{\tau}} \end{aligned}$$

and Φ is the [standard normal cumulative distribution function](#).

Substituting for u , x , and τ , we obtain the value of a call option in terms of the Black–Scholes parameters:

$$V(S, t) = S\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$$

where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}}$$

$$d_2 = d_1 - \sigma\sqrt{T - t}.$$

The formula for the price of a [put option](#) follows from this via [put-call parity](#).

Remarks on notation

The reader is warned of the inconsistent notation that appears in this article. Thus the letter S is used as:

- (1) a [constant](#) denoting the current price of the stock
- (2) a real [variable](#) denoting the price at an arbitrary time
- (3) a [random variable](#) denoting the price at maturity
- (4) a [stochastic process](#) denoting the price at an arbitrary time

Risks

As with all securities, trading options entails the risk of the option's value changing over time. However, unlike traditional securities, the return from holding an option varies non-linearly with the value of the underlier and other factors. Therefore, the risks associated with holding options are more complicated to understand and predict.

In general, the change in the value of an option can be derived from [Ito's lemma](#) as:

$$dC = \Delta dS + \Gamma \frac{dS^2}{2} + \kappa d\sigma + \theta dt$$

where the [greeks](#) Δ , Γ , κ and θ are the standard hedge parameters calculated from an option valuation model, such as [Black-Scholes](#), and dS , $d\sigma$ and dt are unit changes in the underlier price, the underlier volatility and time, respectively.

Thus, at any point in time, one can estimate the risk inherent in holding an option by calculating its hedge parameters and then estimating the expected change in the model inputs, dS , $d\sigma$ and dt , provided the changes in these values are small. This technique can be used effectively to understand and manage the risks associated with standard options. For instance, by offsetting a holding in an option with the amount $-\Delta$ of shares in the underlier, a trader can form a [delta neutral](#) portfolio that is hedged from

loss for small changes in the underlier price. The corresponding price sensitivity formula for this portfolio is:

$$d\Pi = \Delta dS - \Delta dS + \Gamma \frac{dS^2}{2} + \kappa d\sigma + \theta dt = \Gamma \frac{dS^2}{2} + \kappa d\sigma + \theta dt$$

Example

A call option expiring in 99 days on 100 shares of XYZ stock is struck at \$50, with XYZ currently trading at \$48. With future realized volatility over the life of the option estimated at 25%, the theoretical value of the option is \$1.89. The hedge parameters Δ , Γ , κ , θ are (0.439, 0.0631, 9.6, and -0.022), respectively. Assume that on the following day, XYZ stock rises to \$48.5 and volatility falls to 23.5%. We can calculate the estimated value of the call option by applying the hedge parameters to the new model inputs as:

$$dC = (0.5 \cdot 0.439) + \left(\frac{0.5^2}{2} \cdot 0.0631\right) - (0.015 \cdot 9.6) - 0.022 = 0.132$$

Under this scenario, the value of the option increases by \$0.132 to \$2.022, realizing a profit of \$13.20. Note that for a delta neutral portfolio, where by the trader had also sold 44 shares of XYZ stock as a hedge, the net loss under the same scenario would be (\$8.75).

Pin risk

A special situation called [pin risk](#) can arise when the underlier closes at or very close to the option's strike value on the last day the option is traded prior to expiration. The option writer (seller) may not know with certainty whether or not the option will actually be exercised or be allowed to expire worthless. Therefore, the option writer may end up with a large, unwanted residual position in the underlier when the markets open on the next trading day after expiration, regardless of their best efforts to avoid such a residual.

Trading

The most common way to trade options is via standardized options contracts that are listed by various [futures and options exchanges](#).^[11] By publishing continuous, live markets for option prices, an exchange enables independent parties to engage in price discovery and execute transactions. As an intermediary to both sides of the transaction, the benefits the exchange provides to the transaction include:

- fulfillment of the contract is backed by the credit of the exchange, which typically has the highest [rating](#) (AAA),
- counterparties remain anonymous,
- enforcement of market regulation to ensure fairness and transparency, and
- maintenance of orderly markets, especially during fast trading conditions.

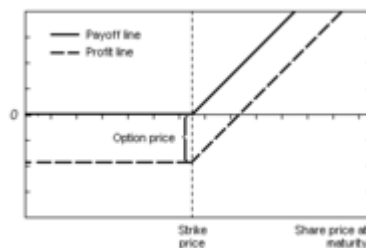
[Over-the-counter](#) options contracts are not traded on exchanges, but instead between two independent parties. Ordinarily, at least one of the counterparties is a well-capitalized institution. By avoiding an exchange, users of OTC options can narrowly tailor the terms of the option contract to suit individual business requirements. In addition, OTC option transactions generally do not need to be advertised to the market and face little or no regulatory requirements. However, OTC counterparties must establish credit lines with each other, and conform to each others clearing and settlement procedures.

With few exceptions,^[12] there are no [secondary markets](#) for [employee stock options](#). These must either be exercised by the original grantee or allowed to expire worthless.

The basic trades or traded stock options

These trades are described from the point of view of a speculator. If they are combined with other positions, they can also be used in hedging.

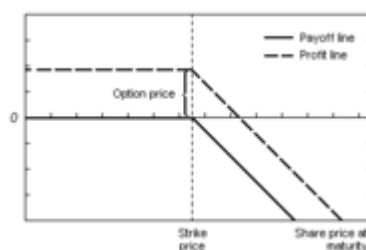
Long Call



Payoffs and profits from a long call.

A trader who believes that a stock's price will increase might buy the right to purchase the stock (a [call option](#)) rather than just buy the stock. He would have no obligation to buy the stock, only the right to do so until the expiration date. If the stock price increases over the exercise price by more than the premium paid, he will profit. If the stock price decreases, he will let the call contract expire worthless, and only lose the amount of the premium. A trader might buy the option instead of shares, because for the same amount of money, he can obtain a larger number of options than shares. If the stock rises, he will thus realize a larger gain than if he had purchased shares.

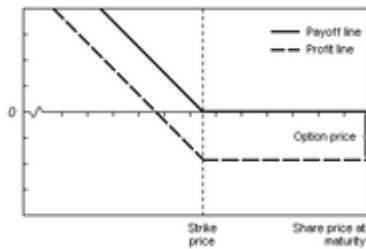
Short Call



Payoffs and profits from a naked short call.

A trader who believes that a stock price will decrease can short sell the stock or instead sell a call. Both tactics are generally considered inappropriate for small investors. The trader selling a call has an obligation to sell the stock to the call buyer at the buyer's option. If the stock price decreases, the short call position will make a profit in the amount of the premium. If the stock price increases over the exercise price by more than the amount of the premium, the short will lose money, with the potential loss unlimited.

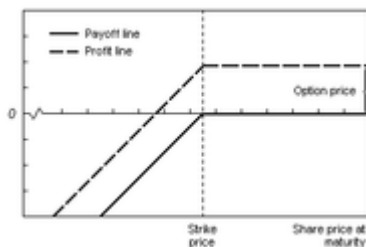
Long Put



Payoffs and profits from a long put.

A trader who believes that a stock's price will decrease can buy the right to sell the stock at a fixed price. He will be under no obligation to sell the stock, but has the right to do so until the expiration date. If the stock price decreases below the exercise price by more than the premium paid, he will profit. If the stock price increases, he will just let the put contract expire worthless and only lose his premium paid.

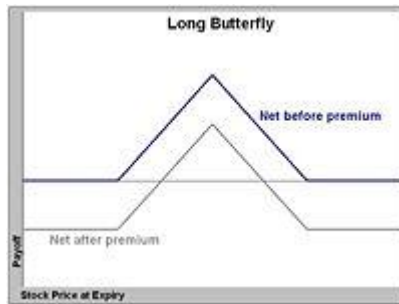
Short Put



Payoffs and profits from a naked short put.

A trader who believes that a stock price will increase can buy the stock or instead sell a put. Shorting puts is generally considered inappropriate for small investors. The trader selling a put has an obligation to buy the stock from the put buyer at the put buyer's option. If the stock price increases, the short put position will make a profit in the amount of the premium. If the stock price decreases below the exercise price by more than the amount of the premium, the short will lose money, with the potential loss being up to the full value of the stock.

Option strategies



Payoffs from buying a butterfly spread.



Payoffs from selling a straddle.

Combining any of the four basic kinds of option trades (possibly with different exercise prices and maturities) and the two basic kinds of stock trades (long and short) allows a variety of options strategies. Simple strategies usually combine only a few trades, while more complicated strategies can combine several.

Strategies are often used to engineer a particular risk profile to movements in the underlying security. For example, buying a butterfly spread (long one X1 call, short two X2 calls, and long one X3 call) allows a trader to profit if the stock price on the expiration date is near the middle exercise price, X2, and does not expose the trader to a large loss.

Selling a [straddle](#) (selling both a put and a call at the same exercise price) would give a trader a greater profit than a butterfly if the final stock price is near the exercise price, but might result in a large loss.

Historical uses of options

Contracts similar to options are believed to have been used since ancient times. In the real estate market, call options have long been used to assemble large parcels of land from separate owners, *e.g.* a developer pays for the right to buy several adjacent plots, but is not obligated to buy these plots and might not unless he can buy all the plots in the entire parcel. Film or theatrical producers often buy the right — but not the obligation — to dramatize a specific book or script. Lines of credit give the potential borrower the right — but not the obligation — to borrow within a specified time period.

Many choices, or embedded options, have traditionally been included in [bond](#) contracts. For example many bonds are convertible into common stock at the buyer's

option, or may be called (bought back) at specified prices at the issuer's option. [Mortgage](#) borrowers have long had the option to repay the loan early.

Privileges were options sold over the counter in nineteenth century America, with both puts and calls on shares offered by specialized dealers. Their exercise price was fixed at a rounded-off market price on the day or week that the option was bought, and the expiry date was generally three months after purchase. They were not traded in secondary markets.

See also

- [American Stock Exchange](#)
- [Chicago Board Options Exchange](#)
- [Eurex](#)
- [Euronext.liffe](#)
- [International Securities Exchange](#)
- [NYSE Arca](#)
- [Options Industry Council](#), sponsored by all 6 options exchanges

[Philadelphia Stock Exchange](#)

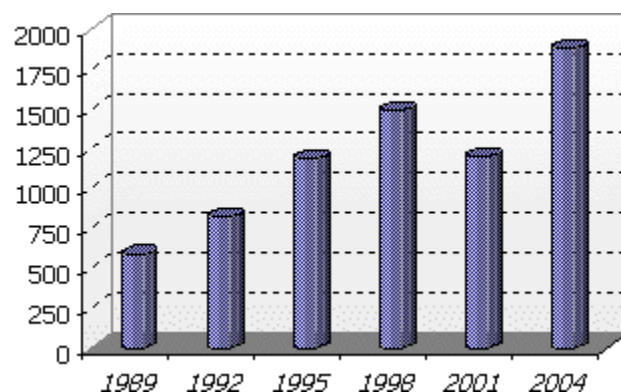
Foreign Exchange Markets

The **foreign exchange (currency or forex or FX) market** exists wherever one [currency](#) is traded for another. It is by far the largest financial market in the world, and includes trading between large banks, [central banks](#), currency [speculators](#), [multinational corporations](#), [governments](#), and other [financial markets](#) and institutions. The average daily trade in the global forex markets currently exceeds [US\\$ 2 trillion](#). Retail traders (individuals) are a small fraction of this market and may only participate indirectly through [brokers](#) or banks.

Market size and liquidity

The foreign exchange market is unique because of:

- its trading volume,
- the extreme [liquidity](#) of the market,
- the large number of, and variety of, traders in the market,
- its geographical dispersion,
- its long trading hours - 24 hours a day (except on weekends).
- the variety of factors that affect [exchange rates](#),



According to the [BIS](#) ^[1], average daily turnover in traditional foreign exchange markets was estimated at \$1,880 billion. Daily averages in April for different years, in billions of US dollars, are presented on the chart below:

This \$1.88 trillion in global foreign exchange market "traditional" turnover was broken down as follows:

- \$621 billion in [spot](#) transactions
- \$208 billion in [outright forwards](#)
- \$944 billion in [forex swaps](#)
- \$107 billion estimated gaps in reporting

In addition to "traditional" turnover, \$1.26 trillion was traded in [derivatives](#).

Exchange-traded forex [futures contracts](#) were introduced in 1972 at the [Chicago Mercantile Exchange](#) and are actively traded relative to most other futures contracts. Forex futures volume has grown rapidly in recent years, but only accounts for about 7% of the total foreign exchange market volume, according to The [Wall Street Journal Europe](#) (5/5/06, p. 20).

Average daily global turnover in traditional foreign exchange market transactions totaled \$2.7 trillion in April 2006 according to IFSL estimates based on semi-annual London, New York, Tokyo and Singapore Foreign Exchange Committee data. Overall turnover, including non-traditional foreign exchange derivatives and products traded on exchanges, averaged around \$2.9 trillion a day. This was more than ten times the size of the combined daily turnover on all the world's equity markets. Foreign exchange trading increased by 38% between April 2005 and April 2006 and has more than doubled since 2001. This is largely due to the growing importance of foreign exchange as an asset class and an increase in fund management assets, particularly of hedge funds and pension funds. The diverse selection of execution venues such as internet trading platforms has also made it easier for retail traders to trade in the foreign exchange market. ^[2]

Because foreign exchange is an [OTC](#) market where brokers/dealers negotiate directly with one another, there is no central exchange or clearing house. The biggest geographic trading centre is the UK, primarily London, which according to IFSL estimates has increased its share of global turnover in traditional transactions from 31.3% in April 2004 to 32.4% in April 2006.

The ten most active traders account for almost 73% of trading volume, according to The [Wall Street Journal Europe](#), (2/9/06 p. 20). These large international banks continually provide the market with both bid (buy) and ask (sell) prices. The [bid/ask spread](#) is the difference between the price at which a bank or [market maker](#) will sell ("ask", or "offer") and the price at which a market-maker will buy ("bid") from a wholesale customer. This spread is minimal for actively traded pairs of currencies, usually only 0-3 [pips](#). For example, the bid/ask quote of EUR/USD might be 1.2200/1.2203. Minimum trading size for most deals is usually \$100,000.

These spreads might not apply to retail customers at banks, which will routinely mark up the difference to say 1.2100 / 1.2300 for transfers, or say 1.2000 / 1.2400 for banknotes or travelers' checks. Spot prices at market makers vary, but on EUR/USD are usually no more than 3 pips wide (i.e. 0.0003). Competition has greatly increased with pip spreads shrinking on the major pairs to as little as 1 to 2 pips.

Market participants

Source: Euromoney FX survey¹

Top 10 Currency Traders		
% of overall volume, May 2006	Name	% of volume
Rank		
1	Deutsche Bank	19.26
2	UBS AG	11.86
3	Citigroup	10.39
4	Barclays Capital	6.61
5	Royal Bank of Scotland	6.43
6	Goldman Sachs	5.25
7	HSBC	5.04
8	Bank of America	3.97
9	JPMorgan Chase	3.89
10	Merrill Lynch	3.68

Unlike a stock market, where all participants have access to the same prices, the forex market is divided into levels of access. At the top is the inter-bank market, which is made up of the largest investment banking firms. Within the inter-bank market, spreads, which are the difference between the bid and ask prices, are razor sharp and usually unavailable, and not known to players outside the inner circle. As you descend the levels of access, the difference between the bid and ask prices widens. This is due to volume. If a trader can guarantee large numbers of transactions for large amounts, they can demand a smaller difference between the bid and ask price, which is referred to as a better spread. The levels of access that make up the forex market are determined by the size of the “line” (the amount of money with which they are trading). The top-tier inter-bank market accounts for 53% of all transactions. After that there are usually smaller investment banks, followed by large multi-national corporations (which need to hedge risk and pay employees in different countries), large hedge funds, and even some of the retail forex market makers. According to Galati and Melvin, “Pension funds, insurance companies, mutual funds, and other institutional investors have played an increasingly important role in financial markets in general, and in FX markets in particular, since the early 2000s.” (2004) In addition, he notes, “Hedge funds have grown markedly over the [2001-2004](#) period in terms of both number and overall size” Central banks also participate in the forex market to align currencies to their economic needs.

Trading characteristics

There is no single unified foreign exchange market. Due to the [over-the-counter \(OTC\)](#) nature of currency markets, there are rather a number of interconnected marketplaces, where different currency [instruments](#) are traded. This implies that there is no such thing as *a single* dollar rate - but rather a number of different rates (prices), depending on what bank or market maker is trading. In practice the rates are often very close, otherwise they could be exploited by [arbitrageurs](#).

The main trading centers are in [London](#), [New York](#), [Tokyo](#), and [Singapore](#), but banks throughout the world participate. Currency trading happens continuously throughout the day; as the Asian trading session ends, the European session begins, followed by the US session and then back to the Asian session, excluding weekends.

Top 6 Most Traded Currencies

Rank	Currency	ISO 4217 Code	Symbol
1	United States dollar	USD	\$
2	Eurozone euro	EUR	€
3	Japanese yen	JPY	¥
4	British pound sterling	GBP	£
5-6	Swiss franc	CHF	-
5-6	Australian dollar	AUD	\$

Factors affecting currency trading

Although exchange rates are affected by many factors, in the end, currency prices are a result of supply and demand forces. The world's currency markets can be viewed as a huge melting pot: in a large and ever-changing mix of current events, [supply](#) and [demand](#) factors are constantly shifting, and the price of one currency in relation to another shifts accordingly. No other market encompasses (and distills) as much of what is going on in the world at any given time as foreign exchange.

Supply and demand for any given currency, and thus its value, are not influenced by any single element, but rather by several. These elements generally fall into three categories: [economic](#) factors, [political](#) conditions and [market psychology](#).

Economic factors

These include economic policy, disseminated by government agencies and [central banks](#), economic conditions, generally revealed through economic reports, and other [economic indicators](#).

Economic policy comprises government [fiscal policy](#) (budget/spending practices) and [monetary policy](#) (the means by which a government's central bank influences the supply and "cost" of money, which is reflected by the level of [interest rates](#)).

Economic conditions include:

Government budget deficits or surpluses: The market usually reacts negatively to widening government [budget deficits](#), and positively to narrowing budget deficits. The impact is reflected in the value of a country's currency.

Balance of trade levels and trends: The trade flow between countries illustrates the demand for goods and services, which in turn indicates demand for a country's currency to conduct trade. Surpluses and deficits in trade of goods and services reflect the competitiveness of a nation's economy. For example, [trade deficits](#) may have a negative impact on a nation's currency.

Inflation levels and trends: Typically, a currency will lose value if there is a high level of [inflation](#) in the country or if inflation levels are perceived to be rising. This is because inflation erodes [purchasing power](#), thus demand, for that particular currency.

Economic growth and health: Reports such as gross domestic product ([GDP](#)), [employment](#) levels, [retail sales](#), [capacity utilization](#) and others, detail the levels of a country's [economic growth](#) and health. Generally, the more healthy and robust a country's economy, the better its currency will perform, and the more demand for it there will be.

Political conditions

Internal, regional, and international [political](#) conditions and events can have a profound effect on currency markets.

For instance, political upheaval and instability can have a negative impact on a nation's economy. The rise of a political faction that is perceived to be fiscally responsible can have the opposite effect. Also, events in one country in a region may spur positive or negative interest in a neighboring country and, in the process, affect its currency.

Market psychology

[Market psychology](#) and trader perceptions influence the foreign exchange market in a variety of ways:

Flights to quality: Unsettling international events can lead to a "[flight to quality](#)" - with investors seeking a "[safe haven](#)". There will be a greater demand, thus a higher price, for currencies perceived as stronger over their relatively weaker counterparts.

Long-term trends: Currency markets often move in visible long-term [trends](#). Although currencies do not have an annual growing season like physical commodities, [business cycles](#) do make themselves felt. Cycle analysis looks at longer-term price trends that may rise from economic or political trends. ^[4]