FINANCE Investments V

Lecture 2

In this lecture, I shall address a number of basic issues.

1. What is the return on a security?

2. How would we expect the returns on different classes of securities to behave?

3. What are the most risky classes of assets?

4. What are the least risky classes of assets?

5. How do we measure risk?

6. How would we expect the term structure of interest rates to behave?

See J F Marshall & V K Bonsai, *Financial Engineering* (1992), New York Institute of Finance, Chapter 5.

Rates of Return and Compounding

There are times when we have a series of observations on wealth and we would like to compute the successive holding period yields.

i.e., Suppose we start with $100 of wealth invested in some security with all income from the security immediately reinvested in additional units of the same security. Each month, we make an observation on the value of our position. The end of month wealth for each of the first six months forms a set of observations as shown below:

Month	End of month wealth	Return Relative	HPY (effective) %	HPY (continuously compounded) %
0	100	--	--	--
1	110	1.100	10.000	9.531
2	105	0.9545	-4.545	-4.652
3	95	0.948	-9.524	-10.008
4	115	1.2105	21.053	19.106
5	125	1.0870	8.696	8.338
6	115	0.9200	-8.000	-8.338

Note that the continuously compounded return is a more accurate reflection of returns and subsections of the holding period than the arithmetic discrete return. Take months 4-6:

Month	End of month wealth	Discrete Return	HPY (continuously compounded) %
4	115	--	--
5	125	1.0870	+8.338
6	115	0.9200	-8.338

Discrete

$$\frac{P5}{P4} + \frac{P6}{P5} \neq \frac{P6}{P4}$$

i.e., $\frac{1.087 + 0.92W}{2} = 1.0035$ or 0.35%

Continuously compounded

$$\ln\left(\frac{P5}{P4}\right) + \ln\left(\frac{P6}{P5}\right) = \ln\frac{P6}{P4}$$

$$+8.338\% - 8.338\% = 0$$

The difference depends on the variance of price movements in the sub-periods.

R G Ibbotson & R A Sinquefield, "Stocks, Bonds, Bills and Inflation: Year-by-Year Historical Returns (1926-1974)", *Journal of Business* (1976).

In the original paper, they present year-by-year historical rates of return for five major classes of assets in the US:

1. Common stocks
2. Long-term US government bonds
3. Long-term corporate bonds
4. US Treasury Bills
5. Consumer goods (inflation)

For each asset, we present total rates of return which reflect dividend or interest income as well as capital gains or losses.

In addition to the five basic series, we present seven derived series.

These derived series represent the component parts of assets returns.

They indicate real (inflation adjusted) returns for the first four basic series.

They also include a series measuring the net return from investing in long-term government bonds rather than bills, the net return from investing in common stocks rather than bills,

and the net return from investing in long-term corporate bonds rather than long-term government bonds.

The basic historical series

They produce five basic series, annual returns are estimated by compounding monthly returns - returns are calculated assuming no taxes or transactions costs.

Common stocks

They use the Standard & Poors (S&P) Composite Index. This is a market value weighted index.

Designating common stocks as m, they form monthly returns by

$$Rm_t = \left(Pm_t + Dm,_t\right) \big/ Pm_{t-1} \quad -1 \qquad (1)$$

Where Rm_t is the common stock total return in month t. Pm_t is the index value at the end of month t. Dm_t is the estimated dividends received during month t.

Long-term US Government Bonds

To measure the total returns of long-term US government bonds, they use CRSP data - the aim is to maintain a 20-year term bond portfolio whose returns do not reflect potential tax benefits, impaired negotiability or special redemption, or call privileges.

Monthly returns on government bonds are calculated as:

$$R_{gt} = \left(P_{g,t} + D_{gt}\right) \big/ P_{g,t-1} \quad -1 \qquad (2)$$

where R_{gt} is the long-term government bond total return during month t, P_{gt} is the average between the bid and ask flat price (includes accrued interest).

D_{gt} is the coupon paid during month t, and investment at the end of month t.

Long-term Corporate Bonds

Solomon Brothers has constructed the High-Grade Long-Term Corporate Bond Index for 1969-74. They backdate the index to 1946-68 and for the period 1926-45 use S+P's monthly High-Grade Corporate Composite Index

$$R_{ct} = \left(P_{ct,19-11} + D_{ct}\right) / P_{c,t-1,20} \quad -1 \qquad (3)$$

where R_{ct} is the monthly bond return for a particular series during month t.

$P_{ct,t-1,20}$ is the purchase price at the end of month t-1, for the yield series gives a 20-month maturity.

$P_{ct,19-11}$ is the sale price of the bond at the end of month t given 19 years 11 months to maturity.

D_{ct} is the coupon received which is $\frac{1}{12}$ of the annual coupon.

United States Treasury Bills

Constant on index that uses the shortest term bills not less than 1 month in maturity.

$$R_{ft} = P_{ft} / P_{ft-1} \quad -1 \qquad (4)$$

Again, the prices used are the average of the bid and ask.

Inflation

They use the Consumer Price Index to measure inflation (CPI) which is the rate of change of consumer goods prices.

Monthly rates of change are formed by

$$R_{It} = \frac{V_{It}}{V_{I,t-1}} - 1 \tag{5}$$

where V_{It} is the value of the CPI measured during month t.

Holding Period Return Matrices for basic series

At the end of each month n, they form a cumulative wealth relative index V_n for each of the monthly return series.

$$V_n = \prod_{t=1/26}^{n} (1 + R_t) \tag{6}$$

They also compute geometric mean annual returns

$$RG*(T_1, T_2) = \left[\prod_{T=T_1}^{T_2} (1 + R_t) \right]^{1/T_2 - T_1 + 1} \tag{7}$$

Derived Series

Risk Premia

$$R_{p,t} = \frac{1 + Rm_t}{1 + Rf_t} - 1 = \frac{Rm_t - Rf_t}{1 + Rf_t} \tag{8}$$

or a simple approximation

$$R_{pt}^{1} = Rm_t - Rf_t \tag{9}$$

Maturity Premia

$$RL_t = \frac{1 + Rg,_t}{1 + Rf_t} - 1 \tag{10}$$

Default Premia

$$Rd_t = \frac{1 + R_{ct}}{1 + R_{gt}} - 1 \tag{11}$$

Inflation adjusted series

$$Rm_{r,t} = \frac{1 + Rm_t}{1 + RI_t} - 1 \tag{12}$$

$$R_{gr,t} = \frac{1 + R_{g,t}}{1 + RI_t} - 1 \tag{13}$$

$$R_{cr,t} = \frac{1 + R_{c,t}}{1 + RI_t} - 1 \tag{14}$$

$$R_{fr,t} = \frac{1 + R_{f,t}}{1 + RI_t} - 1 \tag{15}$$