**INVESTMENT FINANCE V**

**Lecture 5B**

**D B Keim, "The CAPM and equity return regularities",** ***Financial Analysts Journal*** **(May-June 1986).**

The CAPM has occupied a central position in financial economics since its introduction over 20 years ago.

$$E\left(R_i\right) = R_z\left[E(RM) - RZ\right]\beta_i \qquad (1)$$

**Early evidence**

Numerous studies in the early 70s generally supported the CAPM, although finding the coefficient on beta to be only marginally important in explaining cross-sectional differences in average security returns.

In 1977, Roll raised some legitimate questions about the validity of these tests.

**After-tax effects**

Because in the USA dividend income is subject to a higher marginal tax rate than capital gains, taxable investors should rationally prefer a dollar of pre-tax capital gains to a dollar of dividends. An after-tax CAPM has the following general form:

$$E\left(R_i\right) = a_o + a_1\beta_i + a_2 d_i \qquad (2)$$

where $d_i$ equals the dividend yield for security i, and $a_2$ represents an implicit tax coefficient that is independent of the level of the dividend yield.

The question is whether $a_2$ is reliably positive and consistent with realistic tax rates. (Table (1) reports some of the evidence).

There is however evidence of a non-linear relationship between dividend yields and return. Keim (1985) found this stems largely from the exaggerated occurrence of the effect in January. (See Figure A).

**Ex-dividend price behaviour**

Because the ownership claim to a dividend expires at the close of trading before the ex-dividend day, the price of a dividend-paying stock should drop on the ex-dividend day. In the absence of effective taxes (or of a tax differential between dividends and capital gains), transaction costs and information effects, the price drop should equal the value of the dividend.

Numerous studies have found that the fall in price on the ex-dividend day, on average, is less than the value of the dividend.

**Size effects**

There is evidence of a significant relation between common stock returns and the market value of common equity.

Banz (1981), the first to document this phenomenon, estimated a model over the 1931-1973 period of the following form:

$$E(R_i) = a_o + a_1\beta_i + a_2 S_i \tag{3}$$

where $S_i$ is a measure of the relative market capitalisation (size) of firm i. Banz found a negative statistical association between returns and size of approximately the same magnitude as that between returns and beta (See Figure B).

**Seasonal Size Effects**

Researchers have also examined the time-related patterns of portfolio returns stratified by market capitalisation. Brown, Kleidon and Marsh (1983) found that, when averaged over all months, the size effect reverses itself for sustained periods; in many periods there is a consistent premium for small size, whereas in other (fewer) periods, there is a discount.

The magnitude of the size effect also seems to differ across days of the week and months of the year. Keim and Stambough (1984) found that the size effect is most pronounced on Friday (See Figure C).

The most dramatic seasonal pattern involves the turn of the year. Keim (1983) found that the size effect is concentrated in January (See Figure D). (See Table II for international evidence).

The price-earnings effects

Nicholson (1960) published the first study, which showed that low P/E stocks consistently provided returns greater than the average stocks. Basu (1977) introduced the notion that P/E ratios may explain violations of the CAPM and found that, for his sample of NYSE firms, there was a distinct negative relationship between P/E ratios and average returns in excess of those predicted by the CAPM.

But is the P/E ratio proxying for an underlying and more fundamental variable?

**Inter-relations between effects**

Research has documented a strong cross-sectional relation between abnormal returns and market capitalisation, P/E ratios, and dividend yields.

Few would argue that these findings are entirely independent phenomena.

**Other seasonal patterns**

Average stock returns tend to be higher on Fridays and negative on Mondays - "the weekend effect".

The monthly effect was detected by Ariel (1984) who showed that for 1963-81, the average returns on common stocks on the NYSE and AMEX were positive only for the last day of the month and for days during the first half of the month (See Figure F).

**Implications**

Many of these findings are inconsistent with an investment environment where the CAPM is descriptive of reality and argue for consideration of alternative models of asset pricing.

Can these findings be utilised by portfolio managers?

- no guarantee they will persist
- the costs of implementing strategies designed to capture these phenomena may be prohibitive
- "abnormal" returns are defined in relation to the CAPM
- The model may not be adjusting completely for relevant risks and costs

**(d) Fama's four models of market equilibrium**

**4) Returns conform to a risk-return relationship**

Say we use the Black model

$$\tilde{R}_{er} = X_e \tilde{R}_{zt} + \left(1 - X_e\right) \tilde{R} mt \tag{12}$$

where $X_e$ and $\left(1 - X_e\right)$ on the investment proportions invested in M and Z to get the minimum variance portfolio.

The above can be shown to imply that

$$\tilde{R}_e V = \tilde{R}_{zt} + \beta_{em}\left(\tilde{R}_{mt} - \tilde{R}_{zt}\right) \tag{13}$$

where $\beta_{em} = \dfrac{Cov\left(\tilde{R}_e, \tilde{R}_m\right)}{\sigma^2\left(\tilde{R}_m\right)}$

Then if the market is in equilibrium

$$E\left(\tilde{n}_{it} \middle| \theta_{t-1}\right) = 0 \tag{14}$$

and we can examine the behaviour of returns using either

$$R_{it} = R_{zt} + b_{im}\left(R_{mt} - R_{zt}\right) + n_{it} \tag{15}$$

or use the standard Sharpe/Lintner CAPM.

**Tests of strong-form efficiency**

**Tests of mutual fund performance**

a. Mutual fund managers claim they have professional knowledge and that their fund will out-perform a randomly selected 'unmanaged' portfolio.

b. We need to rank managed fund performance according to some index.

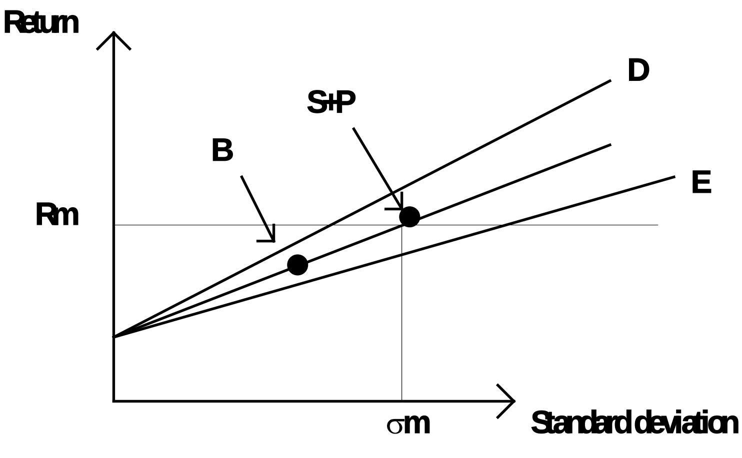
c. A measure of performance is needed for internal requirements to assess manager's performance.

**Sharpe's Performance Index**

$$PIs = \frac{ER_i - r}{\sigma_i}$$

The risky assets' excess return $ER_i - r$ per unit of risk, as measured by the risky assets' standard deviation.

Reward to variability ratio.




D's performance superior, E's performance inferior.

**Tests of strong-form efficiency**

**Treynor's Performance Index**

$$PI_T = \frac{ER_i - r}{\beta_i}$$

Reward to volatility ratio.

$$ER_i = r + \left(ER_m - 1\right)\beta_i$$

$$PI_T = \frac{ER_i - r}{\beta_i} = ER_m - r$$

**Jensen's Performance Index**

$$\overline{R}_i - r = \hat{\alpha}_i + \hat{\beta}_i\left(R_m - r\right)$$

$\hat{\alpha}_i$ measures superior performance.

**Decomposition of excess return - Fama's approach**

Excess return is actual return $\overline{R}_i$ minus the expected return $R_i^*$ where $R_i^*$ is given by the CAPM formula:

$$\overline{R}_i^* = r + \left(\overline{R}_m - r\right)\beta_i$$

$$\overline{R}_i - \overline{R}_i^* = \overline{R}_i - \left[r + \left(\overline{R}_m - r\right)\beta_i\right]$$

The vertical distance in the diagram $\overline{R}_i - \overline{R}_i^*$ is equivalent to Jensen's index.

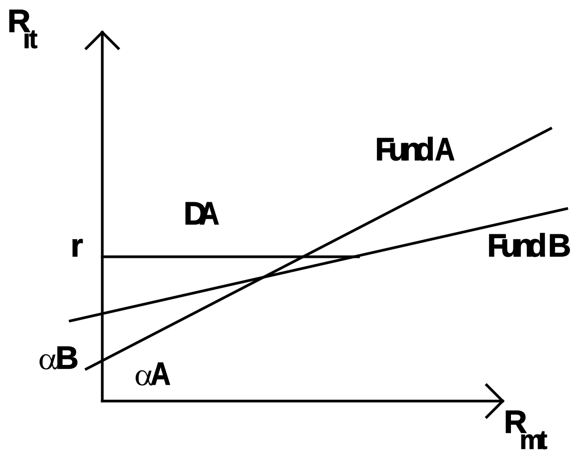
**The relationship between Treynor's and Jensen's performance measures**

Jensen's performance index $PI_j$ is given by the vertical intercept $\hat{\alpha}_i$ from the following time series regression:

$$\frac{\overline{R}_i - r}{\hat{\beta}_i} = \frac{\hat{\alpha}_i}{\beta_i} + \left(\overline{R}_m - r\right)$$

The LHS in Treynor's performance index "Success" by Treynor's index implies $PI_t > \overline{R}_{m-1}$

$\therefore \frac{\hat{\alpha}_i}{\hat{\beta}_i}$ must be positive, since $\beta_i -> 0$ for most funds, we can assert $\hat{\alpha}_i > 0$. Thus, as long as $\beta_i -> 0$, both should register 'success' in similar circumstances. They do not necessarily rank funds performance identically.



**Performance Measures and the Roll Critique of Empirical Studies of the CAPM**

If the market portfolio m is mean-variance efficient, we necessarily obtain in the sample (using ex-post observations) an exact linear relationship between a security's average rate of return and beta

$$\overline{R}_i = r + \left(R_m - e\right)\hat{\beta}_i$$

dividing through by $\beta_i$, we obtain

$$\frac{\overline{R}_i - r}{\hat{\beta}_i} = \frac{\hat{\alpha}_i}{\hat{\beta}_i} + \left(\overline{R}_m - r\right)$$

$\therefore$ the Treynor performance index is constant a cross all securities. Similarly, Jensen's index must vanish, since in the above $\hat{\alpha}_i = 0$ for all risk assets.

In reality, ex-post data revel that various portfolios have performance indexes which are far from identical. According to Roll's analysis, this means that the market portfolio proxy used was mean-variance inefficient.

In practice, most investors do not hold fully diversified portfolios. Their appropriate risk index is neither variance $\sigma_i^2$ nor the beta $\beta$, but some combination of the two. Investors may perceive the 'beta' as a proxy for the true risk when beta is calculated against some composite market index.

**"Does the Stock Market rationally reflect Fundamental Values?", L H Summers (1986)**

The paper examines the power of statistical tests commonly used to evaluate the efficiency of speculative markets. It shows that these tests have very low power. Market valuations can differ substantially and persistently from the rational expectation of the present value of cash flows without leaving statistically discernible traces in the pattern of ex-post returns.

Summers points out that despite the widespread allegiance to the notion of market efficiency, a number of authors have suggested that certain asset prices are not rationally related to economic realities.

Modigliani and Cohn (1979) suggest that the stock market is undervalued because of the existence of inflation illusion. Shiller (1979) (1981) suggests that both bond and stock prices are more volatile than can be justified on the basis of real economic events.

Summers argues that the existing evidence does not establish that financial markets are efficient in the sense of rationally reflecting fundamentals.

- The types of statistical tests that have been used have essentially no power against at least one interesting alternative hypothesis to market efficiency.

The results call into question - suggests Summers the theoretical as well as empirical underpinnings of the EMH.

**Defining market efficiency**

It is assumed that the required expected rate of return on the security is equal to a constant r, which is known with certainty. Standard tests of efficiency are joint tests of efficiency and a model specifying expected returns.

Assume the security in question yields a sequence of cash flows $D_t$. These may be thought of as dividends if the security is a stock, or coupons if the security is a bond.

One statement of the hypothesis of market efficiency holds that:

$$P_t = P_t^* = E\left[\left(\sum_{s=t}^{\infty} \frac{Ds}{(1+r)^{s-t}}\right)\Big|\prod{}_t\right] \tag{1}$$

where $\prod_t$ represents the set of information available to market participants at time t. Equation (1) is mathematically equivalent to the statement that, for all t:

$$P_t = E\left(\frac{P_{t+1}}{1+r}\right) + E\left(D_t\right) \tag{2}$$

or the equivalent statement that

$$E(Rt) = E\left(\frac{P_{t+1}}{P_t} - 1 + \frac{(1+r)t}{P_t} = r\right) \tag{3}$$

where the information set in equations (2) and (3) is taken to be $\prod_t$.

Equation (3) also implies that:

$$R_t = r + et \tag{4}$$

where et is serially uncorrelated and orthogonal to any element of $\prod_t$. Market efficiency is normally tested by adding regression drawn from $\prod_t$ to (4) and testing the hypothesis that their coefficients equal zero, or by testing the hypothesis that et follows a white noise process.

The former represent tests of 'semi-strong' efficiency while the latter are tests of 'weak' efficiency.

**Tests of market efficiency**

Failure to reject a hypothesis is not equivalent to its acceptance. Experiments can falsify a theory by contradicting one of its implications.

**Summers (1986)**

The inability of a body of data to reject a scientific theory does not mean that the tests prove, demonstrate or even support its validity.

Evaluation of any test of a theory requires specification of an alternative hypothesis. An alternative hypothesis to market efficiency holds that:

$$\begin{aligned} P_t &= P_t^* + u_t \\ u_t &= \alpha u_{t-1} + v_t \end{aligned} \qquad (5)$$

where lower-case letters indicate logarithms and $u_t$ and $v_t$ represent random shocks.

This hypothesis implies expectation of the present value of future cash flows by a multiplicative factor approximately equal to (1+ut). The deviations are assumed to follow a first order auto regressive process.

The deviations tend to persist but do not grow forever, so 0≤α≤1.

For simplicity, it is assumed that $u_t$ and $v_t$ are uncorrelated with $e_t$ at all frequencies.

Adopting the approximation that $\ln(1+u_t) = ut$

and that $\frac{\mathrm{Div_t}}{\mathrm{P_t}} \cong \frac{\mathrm{Div_t}}{\mathrm{P_t^*}}$

Equations (3), (4) and (5) imply that excess returns $Z_t = \left(R_t - r\right)$ follow on ARMA (Auto Regressive Moving Average) process.

That is

$$\mathrm{Z_t = \alpha Z_{t-1} + e_t - \alpha e_{t-1} + V_t - V_{t-1}} \tag{6}$$

Equation (6) can be used to calculate the variance and the autocorrelation of $\mathrm{Z_t}$. These calculations yield:

$$\sigma_z^2 = 2(1-\alpha)\sigma_u^2 + \sigma_e^2 \tag{7}$$

$$\rho_k = \frac{-\alpha^{k-1}(1-\alpha)^2 \sigma_u^2}{1(1-\alpha)\sigma_u^2 + \sigma_e^2} \tag{8}$$

where $\rho_k$ denotes the kth-order correlation.

NB: The model predicts that the $\mathrm{Z_t}$ should display negative serial correlation.

**Weak-form tests of market efficiency**

The "power" of weak-form tests of market efficiency can now be evaluated. These tests involve evaluating the hypothesis that

$\rho_k = 0$.

Table 1 presents the theoretical first order autocorrelation for various parameter combinations.

Suppose one is interested in testing market efficiency, using aggregate data on monthly stock market returns over a 50-year period. With 600 observations, the estimated autocorrelations have a standard error of $1/\sqrt{597} \cong 0.42$ on the null hypothesis of zero autocorrelation.

Suppose that $\sigma_z^2$ = .08 so that the standard deviation of the market's error in valuation is close to 30 per cent, and that $\alpha$ = .98.

These assumptions, along with the observation that $\sigma_z^2 \cong .004$, imply, using (7), that $\sigma_e^2 \cong .001$. Equation (8) implies that the theoretically expected value of $\rho_1$ is -0.008. Thus, in this example, the data lack the power to reject the hypothesis of market efficiency, even though market valuations frequently differ from the rational expectation of the present value of future cash flows by more than 30 per cent.

5000 years of data would be required to have a 50 per cent chance of rejecting the null hypothesis.

**Tests of semi-strong efficiency**

Equation (5) implies that the expected excess return should be negative when $P+ > P_t^*$. This reflects the assumed tendency of market prices to return to the rational expectation of the present value of future cash flows. Are the expected excess returns large enough to be recognisable? Given the assumptions made,

$$E\left(Z_t\right) = -(1-\alpha)u_{t-1} = (1-\alpha)\left(P_t^* - P_t\right) \qquad (9)$$

In the previous example with α=.98 and σu = .28, (9) implies that when the market was undervalued by one standard deviation, the expected excess monthly return would be (.02) x (.28) = .0056. This contrasts with a standard deviation of monthly returns of 0.6.

How much data would it take for these excess returns to be statistically discernible? Suppose that the regression equation

$$Z_t = a + b\left(P_t^* - P_t\right) + nt \qquad (10)$$

Equation (9) implies $E\left(\hat{b}\right) = (1-\alpha)$. The standard error of $\hat{b}$ can be calculated from the expression

$$\sigma^2 \hat{b} = \frac{\sigma_n^2}{n\sigma_u^2} \qquad (11)$$

In the example above, $\sigma b \cong .01$. This implies that the hypothesis of market efficiency would not be rejected at the 5 per cent level, with probability of one-half.