FINANCE ECF5103

LECTURE 2

The Gordon Growth Model

Assume dividends grow at a constant rate g. The constant discount rate, or required rate of return for a stock of this risk level is k.

$$Po = \frac{d}{(1+k)} + \frac{d(1+g)}{(1+k)^2} + \frac{d(1+g)^2}{(1+k)^3} + ... + \frac{d(1+g)^{n-1}}{(1+k)^n} \qquad (1)$$

The stock price Po is the present value of future dividends. To sum this series multiply through by $\frac{(1+k)}{(1+g)}$.

$$Po\frac{(1+k)}{(1+g)} = \frac{d}{(1+g)} + \frac{d}{(1+k)} + \frac{d(1+g)}{(1+k)^2} + ...\frac{d(1+g)^{n-2}}{(1+k)^{n-1}} \qquad (2)$$

If we subtract (1) from (2), we have

$$Po\frac{(1+k)}{(1+g)} - Po = \frac{d}{(1+g)} - \frac{d(1+g)^{n-1}}{(1+k)^n}$$

but as $n \rightarrow \infty \ \frac{\mathrm{d}(1+\mathrm{g})^{n-1}}{(1+u)^n} \rightarrow 0$

$\therefore$

$$Po\frac{(1+k-1-g)}{1+g} = \frac{d}{(1+g)}$$

$\therefore$

$$Po\frac{(k-g)}{(1+g)} = \frac{d}{(1+g)}$$

$\therefore$

$$Po = \frac{d}{(k - g)} \qquad (3)$$

In the Gordon growth model k>g,otherwise the share would have an infinite price.

Alternatively, we could write

$$\mathrm{k} = \frac{\mathrm{d}}{\mathrm{p}} + \mathrm{g} \qquad (4)$$

Defining the required rate of return in (4) above.

R J Shiller, "Do Stock prices move too much to be justified by subsequent changes in dividends?", ***AER*** **(June 1981)**

Uses the dividend discount model as the basis of his paper. The implication is that real stock prices equal the present value of rationally expected future real dividends discounted by a constant real discount rate.

The efficient markets model implies that movement in stock prices is driven by new information about future dividends.

Volatility of stock prices - is it too great to be attributed to any objective new information?

See Shiller's Figure 1, which plots a stock price index p_t and an ex-post rational counterpart $p*_t$.

The stock price index p_t is the real Standard and Poor's Composite Stock Price Index (Detrended by dividing by a factor proportional to the long-run exponential growth path).

$p*_t$.is the present discounted value of the actual subsequent real dividends (also as a proportion of the same long-run growth path).

Shiller argues that it is obvious from his diagram that the stock market decline beginning in 1929 could not be rationalised in terms of subsequent dividends.

The efficient markets model can be described as asserting that

$$p_t = E_t\left(p*_t\right)$$

i.e., p_t is the mathematical expectation conditional on all information available at time t of $p*_t$ (the optimum forecast).

One can define forecast error as:

$$u_t = p*_t - p_t$$

The forecast error u_t must be uncorrelated with the forecast, i.e., the covariance between u_t and p_t must be zero, otherwise it would be possible to improve upon the forecast.

The variance of the sum of two uncorrelated variables is the sum of their variances

$$\text{Var (p*)} = \text{Var (u)} + \text{Var (p)}$$

this means that

$$\text{Var (p)} \leq \text{Var (p*)} \qquad (1)$$

or σ (p) ≤ σ (p*)

Shiller claims that it is obvious that this inequality is violated dramatically in his figures (1) and (2). **The simple efficient markets model**.

According to the above, the real price p_t of a share at the beginning of time period t is given by:

$$p_t = \sum_{k=0}^{\infty} \gamma^{k+1} E_t D_{t+k} \qquad 0 < \gamma < 1 \qquad (2)$$

Where D_t is the real dividend paid at the end of time t.

E_t is the expectation conditional on the information available at time t, and γ is the constant real discount factor.

$\gamma = 1/1+r$ where r is the constant real interest rate.

The one period holding return

$$H_t \equiv \left(\Delta p_t + D_t\right)/\, p_t$$

The model (2) has the property that

$$E_t\left(H_t\right) = r$$

The model (2) can be restated in terms of series as a proportion of the long-run growth factor

$$p_t = P_t \big/ \lambda^{t-\gamma}$$

$$d_t = D_t \big/ \lambda^{t+1-\gamma}$$

where the growth factor is $\lambda^{t-\gamma} = \left(l + g\right)^{t-\gamma}$

g is the rate of growth and γ is the base year.

Dividing (2) by $\lambda^{t-\gamma}$ and substituting, one finds

$$p_t = \sum_{k=0}^{\infty} \left(\lambda\gamma\right)^{k+1} E_t d_{t+k}$$

$$= \sum_{K=0}^{\infty} \bar{\gamma}^{K+1} E_t d_{t+k} \qquad (3)$$

The growth rate g must be less than the discount rate r if (2) is to be given a finite price, and hence

$\bar{\gamma} = \lambda\gamma < 1$ and defining $\bar{\mathbf{r}}$ by

$\bar{\gamma} \equiv 1/(1+\bar{r})$ the discount rate appropriate for the p_t and d_t series is $\bar{r} > 0$. This discount rate $\bar{\mathbf{r}}$ turns out to be the mean dividend divided by the mean price.

i.e., $\bar{r} = E(d)/E(r)$

We may also write the model in terms of the ex-post rational price $p*_t$. That is, $p*_t$ is the present value of actual subsequent dividends.

$p_t = E_t(p*_t)$

where $p*_t = \Sigma\bar{\gamma}^{k+1}d_{t+k}$

Since the summation extends to infinity, we never observe $p*_t$ without some error.

We choose an arbitrary value for $p*_t$, p* for 1979 was set at the average detrended real price over the sample. Then we may determine $p*_t$ recursively, by

$p*_t = \gamma(p*_{t+1} + d_t)$ working back from the terminal date.

We have seen that measures of stock price volatility over the past century appear to be far too high, 5-13 times too high - to be attributed to new information about real dividends - if uncertainty about real dividends is measured by the sample standard deviations of real dividends about their long-run essential growth path.

Allan Kleidon (1986), "Variance bounds tests and stock price valuation models"

Kleidon begins by noting that a fundamental problem encountered in testing rational expectations models is the well-known identification issue: if the implications of a particular model are not supported empirically, then is it the fault of the assumptions of market efficiency and rational expectations, the fault of the particular model being tested, or both?

$$p_t = \frac{\sum_{\tau=1}^{\infty} E\left(d_{t+\tau} | \theta_t\right)}{\left(1+r\right)^{\tau}} \tag{1}$$

where r is an assumed constant discount rate, d_t is dividends in time t, and $(X|\theta)$ denotes the conditional distribution of the random variable X given the information θ.

The perfect foresight price is defined as

$$p^*_t = \frac{\sum_{\tau=1}^{\infty} d_{t+\tau}}{\left(1+r\right)^{\tau}} \tag{2}$$

a comparison of (1) and (2) shows

$$p_t = E\left(p^*_t | \theta_t\right) \tag{3}$$

which forms the basis for the variance bound

$$Var\left(p_t\right) \le Var\left(p^*_t\right) \tag{4}$$

The logic behind the bound is the simple and general notion that the variance of the conditional mean of the distribution is less than that of the distribution itself. Since the price p_t is a forecast of $p*_t$ the variance of the forecast p_t should be less than that of the variable being forecast.

Figure 1 plots Standard and Poor's Annual Composite Stock Price Index 1926-1979, $p*_t$ calculated by the following recursion implied by definition (2)

$$p*_t = \frac{p*_{t-1} + d_{t+1}}{(1+r)} \qquad (5)$$

However, figure 2 is based on simulated data that by construction are generated by the rational valuation model (1).

Kleidon argues that the fundamental flaw in the current interpretation of inequality (4) is that it is essentially a cross-sectional relation across different economies but figures (1) and (2) give time series plots for a single economy.

The bound (4) is derived with respect to different values of p* that differ from each other at date t.

If future realisations of dividends are unexpectedly good, the realised value of $p*_t$ will be greater than what is expected at t, which by (3) is p_t, if the future is unexpectedly bad, $p*_t$ is less than p_t.

Kleidon argues that the problem with using real data is that ex-post we can observe only one of the ex-ante possible economies, and so we cannot look across different values of $p*_t$, each corresponding to a different economy, to see if they are correct.

Since by construction p^*_t is always calculated using all realised future dividends, there are no unexpected changes in dividends with implications for changes in p^*_t as there are for prices. In fact, the ex-post return from both dividends and capital gains will always equal the discount rate r for the p^*_t series, by definition (2) - therefore we get a smooth plot.

Variance bounds tests and short-term variances

Equation (3) implies

$$p^*_t = p_t + \varepsilon_t$$

where $E(\varepsilon_t | p_t) = 0$ by rational expectations.

Clearly Variance $p^*_t \geq$Variance (p_t)

which gives the variance bound (4) in terms of *unconditional variances* of p^*_t and p_t. This illustrates the essentially cross-sectional nature of the bound.

At any date t the realised information set θ_t restricts the possible economies that may occur.

The possible values of the present value of dividends in these economies are given by the *conditional distribution* $(p^*_t | \theta_t)$ with expectation p_t.

Each possible realisation of θ_t implies a (possibly different) conditional distribution for p^*_t, including the conditional expectation p_t. Integration over all possible economies results in the distribution of prices with the variance var (p_t) used in the bound (4) and the unconditional distribution of p^*_t.

This argument also applies to distributions other than the unconditional distributions which result when all possible realisations of θ_t are considered, i.e., knowledge of θ_{t-1} may restrict the possible economics at t relative to the total set.

More generally (7) implies

$$Var(p^*_t|\theta_{t-k}) = Var(p_t|\theta_{t-k}) + Var(\varepsilon_t|\theta_{t-k})$$
$$\geq Var(p_t|\theta_{t-k}), \text{ k} = 1,...,\infty \quad (8)$$

where information at t-k is included in the information set at t, and rational expectations require that $\text{Cov}(\varepsilon_t, p_t\ \theta_{t-k}) = 0$.

The inequalities in (8) are useful if conditional variances $(K < \infty)$ are defined but unconditional variances $(k = \infty)$ are not.

Kleidon clams that the confusion in the interpretation of time series plots of price and p^*_t stems from comparing the conditional variance of price $Var(p_t|p_{t-k})$ with an inappropriate conditional variance of p^*_t, $Var(p_t|p_{t-k})$ *which does not limit the conditioning information to information available to traders at t-k.*

Kleidon's example

Consider the following process

$$d_t = \alpha d_{t-1} + n_t \quad (9)$$

where n_t is $\text{i.i.d}(0, \sigma^2{}_n)$. Then the following results hold - Proposition 1. If prices are set by (1) and information comprises current and past dividends given by (9), then:

$$p_t = bd_t$$
$$= \alpha p_{t-1} + bn_t \quad (10)$$

where

$$b = \frac{\alpha}{(1 + r - \alpha)}$$

This process includes both stationary dividends $(\alpha < 1.0)$ and non-stationary random walk dividends $(\alpha = 1.0)$.

Kleidon then proceeds to derive the variances of conditional distributions $(p_t|\theta_{t-k})$ and $(p*_t|\theta_{t-k})$, where θ_{t-k} is limited to current and past dividends.

Kleidon's resolution of the paradox

He argues that of particular interest in the behaviour of the $\text{var}(p*_t|p*_{t-k})$ relative to $Var(P_t|p_{t-k})$ which determines the relative smoothness of the series. Both equal 0 at k=0, and for some value k, it must be the case that $Var(p*_t|p*_{t-k}) > Var(P_t|p_{t-k})$, since we know that eventually the unconditional variances of $p*_t$ and p_t satisfy this inequality.

The key result is that the short-term variances show the opposite result. For small k, we see:

$$Var(p*_t|p*_{t-k}) < Var(p_t|p_{t-k})$$

and this can hold for quite large k depending upon the parameter α in the dividend process. This implies that plots of $p*_t$ should show greater smoothness than in the price series and is no evidence for violation of the bound.

"Dividend variability and variance bounds tests for the rationality of stock market prices", T A Marsh and R C Merton, *American Economic Review* (1986)

The strength of Shiller's conclusions is derived from three elements:

1. The apparent robustness of the variance bounds methodology.
2. The length of the data sets used in the analysis - one set has over 100 years of dividend and stock price data.
3. The magnitude of the empirical violation of his upper bound for the volatility of rational stock prices.

Shiller in essence relies on (2) and (3) above to argue that his rejection of the efficient market model cannot be explained away by 'mere' sampling error.

In their paper, Marsh and Merton focus on (1) and conclude that the variance bound methodology is a wholly unreliable means of testing stock market rationality.

To support their claim, they present an alternative variance bound test which has the feature that of necessity observed prices will be judged rational if they fail the Shiller test.

The paradox arises from differences in assumptions about the underlying stochastic processes used to describe the evolution of dividends and rational stock prices.

The key assumptions underlying Shiller's test can be summarised:

S1. Stock prices reflect investor beliefs which are rational expectations of future dividends.

S2. The "real" expected rate of return on the stock market r, is constant over time.

S3. Aggregate real dividends on the stock market (D(t)) can be described by a finite-variance stationary stochastic process with a deterministic exponential growth trend denoted by g.

Shiller's findings are a rejection of the joint hypothesis of S1, S2 and S3.

Marsh and Merton argue that the Shiller variance bound test is very sensitive to the posited dividend process S3.

Marsh and Merton use a dividend smoothing process based on the Lintner model.

$$\Delta D(r) = gD(t)$$

$$+\sum_{k=0}^{N} \gamma k\left[\Delta E(t-k) - gE(t-k)\right]$$

On the basis of this, they reverse the direction of the Shiller variance bound test.

***QJE*, May 1993**

R B Borsky & J Bradford De Long, "Why does the stock market fluctuate?"

Major long-run swings in the US stock market over the past century are broadly consistent with a model driven by changes in current and expected future dividends in which investors must estimate the time-varying long-run dividend growth rate.

Such an estimated growth rate over the long run resembles a long distributed lag on past dividend growth and is highly correlated with the level of dividends. Prices therefore respond more than proportionately to long-run movements in dividends. The time varying component of dividend growth need not be detachable in the dividend data for it to have large effects on stock prices.

1. They stress that the log dividend process is to a rough approximation a random walk. For such a process, the warranted stock price does move proportionately with dividends.

2. They stress that not only are stock prices not "smoother" than dividends, prices also appear to 'overreact' to long swings in dividends.

3. They explain this follows from their dropping of the assumption that the dividend growth rate has a mean that is constant over time and known to agents throughout the sample.

Given the Gordon Growth

$$P_t = Dt/(r - gt) \tag{1}$$

r

Normally, the estimate gt will resemble a distributed lag on past one-period dividend growth rates with slowly declining weights - it will closely resemble the level of dividends. Since Dt and gt are closely correlated and affect P_t in the same direction, P_t will mirror Dt with an elasticity greater than unity.

The issue is centred on the problem of moving from ex-ante expectations to ex-post realisations.

They argue that the observed price dividend behaviour is what one would see if agents view the dividend process as containing a persistent growth component.

Further issues

1 Assumptions about the revision of expectations

The inequality (1) $\sigma(p) \leq \sigma(p^*)$ was derived using the assumption that the forecast error $u_t = p^*_t - p_t$ is uncorrelated with p_t. However, the forecast error is not serially uncorrelated.

Shiller shows that the maximum variance occurs when innovations in price are perfectly positively correlated, and when information about dividends is revealed in a smooth fashion.

2. High kurtosis and infrequent important breaks in information

It has repeatedly been noted that stock price change distributions show high kurtosis or "fat tails".

This is commonly attributed to a tendency for new information to come in lumps infrequently. High sample kurtosis does not indicate infinite variance if we do not assume, as did Fama (1965) and others, that price changes are drawn from the stable Paretini class of distributions. His model does not suggest that price changes have a distribution in this class.

The model instead suggests that the existence of **moments** for the price series is **implied** by the **existence of moments** for the **dividend series**.

As long as d is jointly stationary with information and has a finite variance, then p, p*, δp and Δp will be stationary and have a finite variance.

3. Dividends or earnings valuation models? Should not make a difference!

4. Time varying real discount rates

If we modify the model to allow real discount rates to vary without restriction through time, then the model becomes untestable. But says that realistic variations in real rates, set by using nominal rates do not explain the massive differences in variances.