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# Variance Bounds Tests and Stock Price Valuation Models

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Previous use of plots of stock prices and “perfect-foresight” prices  $p_t^*$  as evidence of either “excess volatility” or nonconstant discount rates is invalid since by construction  $p_t^*$  will differ from and be much smoother than rational prices if discount rates are constant. Further, prices appear nonstationary, which can account for the previously reported gross violations of variance bounds. Conditional variance bounds that are valid under nonstationarity are not violated for Standard and Poor’s data. The results are consistent with changes in expectations of future cash flows causing changes in stock prices.

## I. Introduction

The question what determines changes in stock prices has long intrigued economists. The suggested answers cover the range from the “animal spirits” of Keynes (1936, p. 161) to models of market efficiency and rational expectations, for example, in Fama (1970*b*). A fundamental problem in testing rational expectation models is the well-known identification issue: If the implications of a particular model are not supported empirically, is it the fault of the assumptions of market efficiency and rational expectations, the fault of the particular model being tested, or both?

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Another possibility is that the model has not been adequately tested either because additional assumptions required to conduct the tests are violated empirically or because the data used simply do not correspond to the theory. It is argued here that these problems are found in much of a recent literature that has led to a resurgence in stated opposition to the belief that stock prices represent a rational valuation of future cash flows. Tobin (1984, p. 26), for example, cites Shiller (1981*b*) as showing that "asset markets [do not] in fact generate fundamental valuations. The speculative content of market prices is all too apparent in their excessive volatility." He continues: "Keynes's classic description of equity markets as casinos where assessments of long-term investment prospects are overwhelmed by frantic short-term guesses about what average opinion will think average opinion will think . . . rings as true today as when he wrote it" (see also Ackley 1983, p. 13; Arrow 1983, p. 12). These are strong statements; however, this paper will argue that they are not justified by the work offered in their support.

The variance bounds literature referred to by Tobin uses a deceptively simple idea to test stock price valuation models based on Miller and Modigliani (1961), with an assumption of constant discount rates. As shown in Miller and Modigliani, there are several equivalent representations in terms of dividends, earnings, and investments. Shiller uses the following dividend model:<sup>1</sup>

$$p_t = \sum_{\tau=1}^{\infty} \frac{E\{d_{t+\tau}|\Phi_t\}}{(1+r)^\tau}, \quad (1)$$

where  $r$  is an assumed constant discount rate,  $d_t$  is dividends in time  $t$ , and  $\{X|\Phi\}$  denotes the conditional distribution of the random variable  $X$  given the information  $\Phi$ . The "perfect-foresight price"  $p_t^{*2}$  is defined as

$$p_t^* \equiv \sum_{\tau=1}^{\infty} \frac{d_{t+\tau}}{(1+r)^\tau}. \quad (2)$$

A comparison of (1) and (2) shows that

$$p_t = E\{p_t^*|\Phi_t\}, \quad (3)$$

which forms the basis for the variance bound

$$\text{var}(p_t) \leq \text{var}(p_t^*). \quad (4)$$

<sup>1</sup> Grossman and Shiller (1981) and Shiller (1981*a*, 1981*b*, 1981*c*) work with this model, while LeRoy and Porter (1981) use the earnings stream approach.

<sup>2</sup> See Shiller (1981*c*, p. 292). The term "perfect-foresight price" is unfortunate since  $p_t^*$  as defined in (2) will not necessarily be the price that would prevail under certainty. See Sec. IV below for further comment.

The logic behind the bound is the simple and general notion that the variance of the conditional mean of a distribution is less than that of the distribution itself. Since the price  $p_t$  is a forecast of  $p_t^*$ , the variance of the forecast  $p_t$  should be less than that of the variable being forecast.

Figure 1 plots Standard and Poor's (1980) annual composite stock price index 1926–79 augmented with the Cowles et al. (1938) common stock index 1871–1925 (the solid line) and  $p_t^*$  calculated from the following recursion implied by definition (2):

$$p_t^* = \frac{p_{t+1}^* + d_{t+1}}{1 + r}, \tag{5}$$

subject to a condition that equates the terminal  $p_T^*$  to the terminal price  $p_T$ . It seems obvious from figure 1 that the bound in (4) is grossly violated, with the consequent implication that prices cannot be set by the model (1). Since (1) implies that changes in price are driven by changes in expectations of future cash flows, it seems reasonable to infer that something else must be causing the large variation in prices. Tobin relies on speculation unrelated to fundamental values.

The data shown in figure 1 were used in Shiller (1981*b*), but similar characteristics are apparent in other data as well. Consider figure 2, which also plots prices  $p_t$  (the solid line) and corresponding  $p_t^*$  series. The relevant characteristics are very similar to those in figure 1.

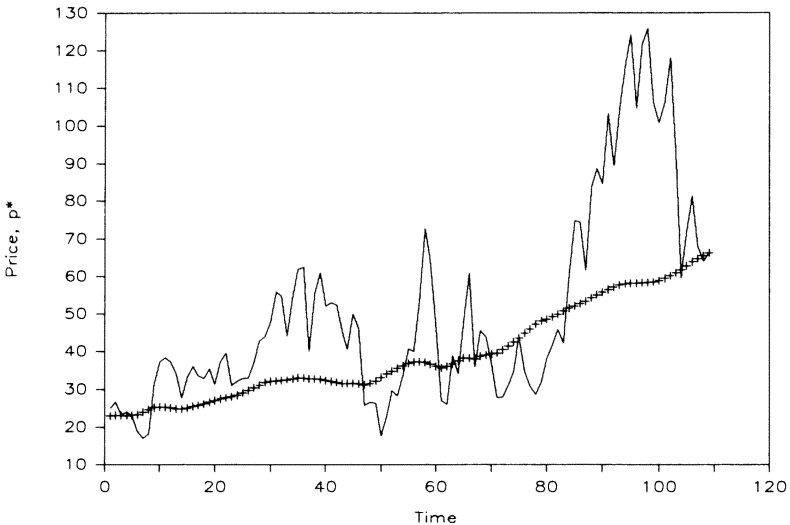


FIG. 1.—Standard and Poor's (real) annual composite stock price index 1926–79 augmented with Cowles Commission common stock index 1871–1925 (solid line) and corresponding perfect-foresight series, including terminal condition  $p_T^* = p_T$ .

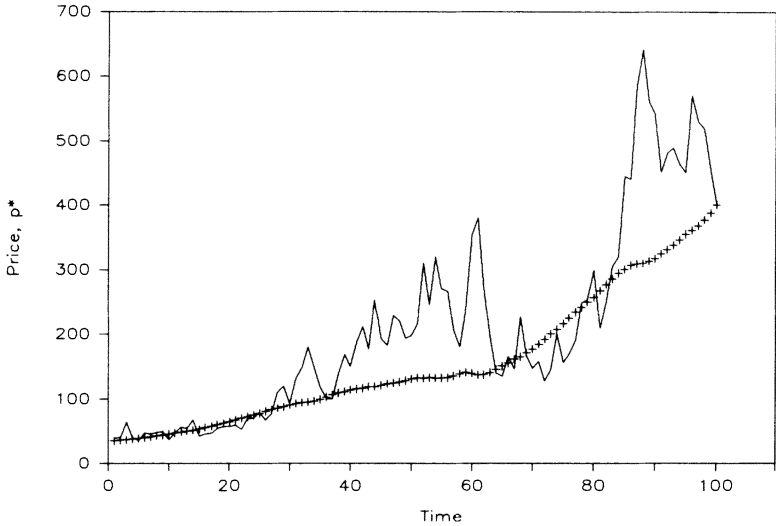


FIG. 2.—Nonstationary (geometric random walk) price series (solid line) and corresponding perfect-foresight series, including terminal condition  $p_T^* = p_T$ .

Again, it seems obvious that the bound (4) is violated and that consequently the valuation model (1) is empirically untenable.

However, such conclusions based on figure 2 are absolutely unfounded. This figure is based not on real data but on simulated data that by construction are generated by the rational valuation model (1). The variance bound (4) is *not* violated, and absolutely nothing can be inferred from the plots about the validity of the model (1).

This seems startling at first glance. Much of the impact of the variance bounds literature has come from the apparent clear violation of the inequality (4) by plots such as figure 1. Indeed, it has been claimed that an inspection of these plots provides such obvious evidence against the inequality (4) and the valuation model (1) that formal empirical tests of (4) need not be relied on (see Shiller 1981a, pp. 4, 7; 1984). Tirole (1985, p. 1085) also claims: "Simply by looking at Figures 1 and 2 in Shiller [1981b], this inequality [i.e., (4)] is not satisfied." This interpretation is clearly false if plots virtually identical to figure 1 can be readily created when (1) holds by construction.

More important, the price process used in figure 2 is not an unusual or artificial construct, but rather is the (geometric) random walk traditionally regarded in finance as an excellent empirical description of the price process in actual data.<sup>3</sup> This paper examines Standard and

<sup>3</sup> For construction details, see Sec. II $\bar{A}$  below, particularly n. 7. Note also that the primary characteristics of time-series plots such as figs. 1 and 2 do not depend on the nonstationarity assumption and are present even in stationary AR(1) processes for prices, as demonstrated in Sec. II below. See Kleidon (1986) for more detail on the stationary case.

Poor's series in some detail and demonstrates empirically that the traditional process used to construct figure 2 is consistent with Standard and Poor's price series in figure 1.

The economic intuition behind the compatibility of plots such as figures 1 and 2 with the variance bound (4) is simple, once one sees it. The fundamental flaw in the current interpretation is that the inequality (4) is essentially a *cross-sectional* relation across different economies, but figures 1 and 2 give time-series plots for a single economy. The bound (4) is derived with respect to values of  $p^*$  that differ from each other at date  $t$  because different realizations of future dividends have different present values at date  $t$ . These different realizations occur across the different economies or worlds that may possibly occur in the future, looking forward from date  $t$ . If future realizations of dividends are unexpectedly good, the realized value of  $p_t^*$  will be greater than what is expected at  $t$ , which by (3) is simply the current price  $p_t$ . If the future is unexpectedly bad,  $p_t^*$  is less than  $p_t$ .

Consider the possible values of  $p^*$  and price that may occur at some particular date  $t$ . If the price  $p_t$  predicts  $p_t^*$ , the theory given by (4) states that there should be greater variation across all possible realizations of  $p_t^*$  than in  $p_t$ . The problem with using real data is that ex post we can observe only one of the ex ante possible economies, and so we cannot look across different values of  $p_t^*$ , each corresponding to a different economy, to see if the theory is correct. We can do this by simulation, however, and it is shown below that precisely the predicted relation across different possible economies holds for the process used to construct figure 2, which is a time-series plot of only one of the ex ante possible outcomes.

Given that we observe only one world in practice, it is important to examine what should be expected in plots of time series of price and  $p^*$  for a single economy. First, note that we would not expect the series to look like each other if there is uncertainty at  $t$  about future dividends since the price  $p_t$  will be the expected value of  $p_t^*$  across possible economies and the ex post value of  $p_t^*$  once the future is revealed will in general differ from its expected value at  $t$ . How much difference will exist between plots of  $p_t^*$  and  $p_t$  depends on the amount of information available when prices are set, and it is shown below that figures 1 and 2 are consistent with a reasonable assumption about information available when Standard and Poor's prices are set.

The second insight, which is crucial to an interpretation of plots such as figures 1 and 2, is that the dividend stream being forecast at dates  $t - 1, t - 2, \dots$  and  $t + 1, t + 2, \dots$  is essentially identical to the stream forecast at  $t$ , and hence the present value of the ex post realizations will be highly correlated. Consequently the time series of  $p_t^*$  will be highly correlated, which translates into the "smooth" time-series path given in figures 1 and 2.

Of course, since  $p_t^*$  depends on information about future dividends not known at  $t$ , it is not part of the information used to set  $p_t$  or, indeed, any other price. At each date the best available information is used to set prices, and as information changes, the price will change. If, for example, the information  $\Phi_t$  comprises current and past dividends, any change in dividends at  $t$  will in general imply changes in all future dividends, and the price will change by the present value of the change in expected dividends. Empirically, changes in dividends tend to persist for a very long time, and so the implied revisions in price can be very large relative to the change in current dividends.

But since by construction  $p_t^*$  is always calculated using all realized future dividends, there are no unexpected changes in dividends with implications for changes in  $p_t^*$  as there are for prices. In fact, the ex post return from both dividends and capital gains will always exactly equal the discount rate  $r$  for the  $p_t^*$  series, by the definition (2). Therefore, the possible change in consecutive values of  $p_t^*$  is limited to the capital gain required to give the ex post return  $r$ , which is another way of stating why the time series  $p_t^*$  can be much smoother than that of price. Consequently, one should expect time-series plots of  $p_t^*$  and  $p_t$  for a single economy to look like figures 1 and 2, even if *across possible economies* the variability of  $p_t^*$  exceeds that of  $p_t$ .

These arguments are established more rigorously in Section II, which demonstrates that plots such as figures 1 and 2 cannot be used to replace more formal tests of the inequality (4). Further, it is clear that, since (4) is derived by considering alternative possible economies, extra assumptions must be made to test (4) using time-series data for only one economy. Section III shows empirically that the traditional assumption in finance of nonstationary (random walk) prices is not rejected for Standard and Poor's series and that the gross violations of (4) currently reported in the literature are consistent with incorrect assumptions of stationarity in the time-series tests conducted. Section III also derives and tests inequalities similar to (4) that are implied by the (geometric random walk) time-series process for prices. It is shown that Standard and Poor's price and dividend data do not violate these bounds. Section IV contains a summary and concluding remarks.

## II. Interpretation of Plots of Price and $p_t^*$

The current interpretation of plots such as figures 1 and 2 is that they demonstrate that prices are not set by the valuation model (1). Although the literature is not always clear on the reasoning, there appear to be two related arguments based on these plots. The first, examined in Section IIA, relies on the undisputed smoothness of a

time-series plot of  $p_t^*$  relative to prices as evidence against (1). Section IIB discusses the second, which attempts to infer the plausibility of the model from the degree of correspondence between the series  $p_t$  and  $p_t^*$ . The argument based on smoothness is clearly a less stringent test than that based on correspondence since two series may be drawn from similar stochastic processes and hence show similar time-series properties, yet not show correspondence between the observations. The conclusion reached here is that neither argument is valid.

#### A. Variance Bounds and "Short-Term Variation"

The characteristic of the time-series plots of price  $p_t$  and  $p_t^*$  that seems most at odds with the claim that  $\text{var}(p_t^*) \geq \text{var}(p_t)$  is the striking "smoothness" of  $p_t^*$  compared with the price series. The current interpretation in the literature is that this is evidence against the inequality. However, this interpretation is incorrect, and in fact the bound does not address the issue of how smooth one time series is compared with the other. The literature has incorrectly identified the variances used in the inequality (4) with smoothness or "short-term variation" in time-series plots of price and  $p_t^*$ .

Examples of this argument occur frequently in the variance bounds literature. For example, Shiller (1981*b*, p. 421) states that "one is struck by the smoothness and stability of the *ex post* rational price series  $p_t^*$  when compared with the actual price series." Grossman and Shiller (1981), in one of the most influential papers using the argument, assume a constant relative risk aversion utility of consumption function,

$$U(c) = \frac{1}{1-A} c^{1-A}, \quad 0 < A < \infty, \quad (6)$$

and calculate (p. 223) the "perfect-foresight stock price"  $p_t^{*1}$  with constant and nonconstant discount rates. Under the assumption that investors know the whole future path of consumption (p. 223), they calculate implied discount rates from (6) for different values of the risk aversion parameter and attempt to infer the parameter value that makes the observed stock price series consistent with market efficiency (p. 224). The risk neutrality case ( $A = 0$ ) gives constant discount rates (assuming constant time preference), and  $p_t^*$  appears much closer to the actual price series for  $A = 4$  (nonconstant rates), at least for the period up to about 1950. Their results are reproduced here as figure 3.

<sup>1</sup> Some papers use the notation  $P_t^*$  and  $P_t$ , as in Grossman and Shiller (1981), while others use the lower-case notation  $p_t^*$  and  $p_t$ , which is used throughout this paper.



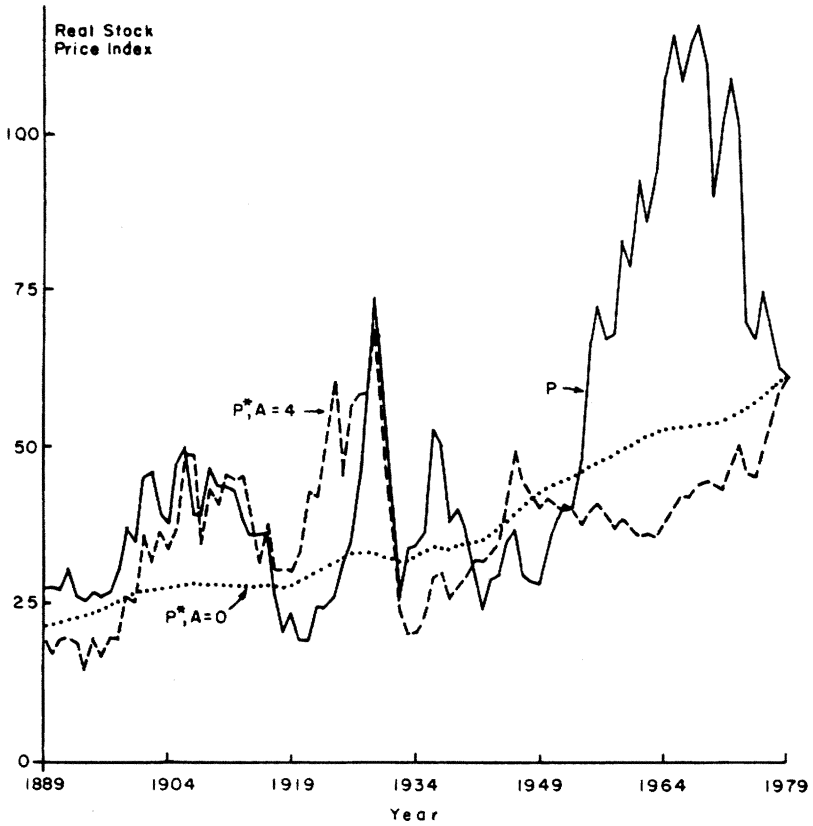


FIG. 3.—Grossman and Shiller's (1981) series of actual and perfect-foresight stock prices, 1889–1979 (reproduced from p. 225). The solid line  $P_t$  is the real Standard and Poor's composite stock price average. The other lines are:  $P_t^*$  (as defined by their expressions 6 and 7), the present value of actual subsequent real dividends using the actual stock price in 1979 as a terminal value. With  $A = 0$  (dotted line) the discount rates are constant, while with  $A = 4$  (dashed line) they vary with consumption.

Grossman and Shiller select the risk aversion parameter  $A = 4$  in figure 3 (1981, p. 224) because of the smoothness of  $p_t^*$  when discount rates are assumed constant: “Notice that with a constant discount factor,  $P_t^*$  just grows with the trend in dividends; it shows virtually none of the *short-term variation* of actual stock prices. The larger  $A$  is, the bigger the variations of  $P_t^*$  and  $A = 4$  was shown here because for this  $A$ ,  $P$ , and  $P^*$  have movements of very similar magnitude” (emphasis added).

It has been shown in figure 2 that  $p_t^*$  is much smoother than price even if the constant discount rate model (1) holds by construction. The primary cause of the confusion shown in the current literature is related to the construction of  $p_t^*$  using ex post information not avail-

able when prices are set. The variance bound (4) is essentially a cross-sectional restriction on the prices that would prevail across different economies at date  $t$ . Tests of the bound using time-series data for a single economy, which are found throughout the literature, require additional strong assumptions beyond those needed to derive (4), and care must be exercised to ensure that the “variances” discussed with respect to time-series data correspond to those in the variance inequality. This section first highlights the cross-sectional nature of the inequality, then shows exactly how the argument in the literature fails.

### 1. Cross-sectional Variance Bounds

The equations used to derive the bound are (1)–(3) above. Equation (3) implies

$$p_t^* = p_t + \xi_t, \tag{7}$$

where  $E\{\xi_t|p_t\} = 0$  by rational expectations. Clearly  $\text{var}(p_t^*) \geq \text{var}(p_t)$ , which gives the variance bound (4) in terms of the unconditional variances of  $p_t^*$  and  $p_t$ . This illustrates the essentially cross-sectional nature of the bound. At any date  $t$  the realized information  $\Phi_t$  restricts the possible economies that may occur, and the possible values of the present value of dividends in those economies are given by the conditional distribution  $\{p_t^*|\Phi_t\}$ , with expectation  $p_t$  by (3). Each possible realization for  $\Phi_t$  implies a (possibly different) conditional distribution for  $p_t^*$ , including the conditional expectation  $p_t$ . Integration over all possible economies results in the distribution of prices with variance  $\text{var}(p_t)$  used in the bound (4) and the unconditional distribution of  $p_t^*$ .

This argument also applies to distributions other than the unconditional distributions that result when all possible realizations of  $\Phi_t$  are considered. For example, knowledge of  $\Phi_{t-1}$  may restrict the possible economies at  $t$  relative to the total set. More generally, (7) implies that

$$\begin{aligned} \text{var}\{p_t^*|\Phi_{t-k}\} &= \text{var}\{p_t|\Phi_{t-k}\} + \text{var}\{\xi_t|\Phi_{t-k}\} \\ &\geq \text{var}\{p_t|\Phi_{t-k}\}, \quad k = 0, \dots, \infty, \end{aligned} \tag{8}$$

where information at  $t - k$  is included in information at  $t$  (traders do not forget), and rational expectations require that  $\text{cov}\{\xi_t, p_t|\Phi_{t-k}\} = 0$ . The inequalities in (8) are clearly useful if conditional variances ( $k < \infty$ ) are defined but unconditional variances ( $k = \infty$ ) are not—for example, for the case of a random walk in prices, which is shown below to be empirically relevant. Further, it is shown below that confusion in interpretation of time-series plots of price and  $p_t^*$  stems from compar-

ing the conditional variance of price,  $\text{var}\{p_t|p_{t-k}\}$ , with an inappropriate conditional variance of  $p_t^*$ ,  $\text{var}\{p_t^*|p_{t-k}^*\}$ , which does not limit the conditioning information to information available to traders at time  $t - k$ .

To illustrate the distinctions, consider the following dividend process (which ignores irrelevant means for current purposes):

$$d_t = \rho d_{t-1} + \eta_t, \quad (9)$$

where  $\eta_t$  is independently and identically distributed (i.i.d.)  $(0, \sigma_\eta^2)$ . Then we have the following proposition.

**PROPOSITION 1.** If prices are set by (1) and information comprises current and past dividends given by (9), then

$$\begin{aligned} p_t &= ad_t \\ &= \rho p_{t-1} + a\eta_t, \end{aligned} \quad (10)$$

where  $a \equiv \rho/(1 + r - \rho)$ .

*Proof.* Follows directly from substitution in (1) for expected future dividends given (9), with simplification of the resulting infinite series. Q.E.D.

This process includes both stationary dividends ( $|\rho| < 1.0$ ) and nonstationary random walk dividends ( $\rho = 1.0$ ). We proceed by giving the variances of the conditional distributions  $\{p_t|\Phi_{t-k}\}$  and  $\{p_t^*|\Phi_{t-k}^*\}$ , where  $\Phi_{t-k}$  is limited to current and past dividends or, equivalently from (10), to  $p_{t-k}$ . The limit as  $k \rightarrow \infty$  gives the unconditional distributions. The variances of the appropriate conditional distributions verify (8), but for the random walk case when  $\rho = 1.0$ , the conditional variances are well defined but the unconditional variances are not.

**PROPOSITION 2.** Assume prices are set by (1) with current and past dividends given by (9) as information. Then

$$\begin{aligned} \text{var}\{p_t|\Phi_{t-k}\} &= \text{var}\{p_t|p_{t-k}\} \\ &= \sigma_\eta^2 a^2 \left( \frac{1 - \rho^{2k}}{1 - \rho^2} \right). \end{aligned} \quad (11)$$

*Proof.* Given the dividend process (9), the result follows directly from (10) conditioned on  $p_{t-k}$  with simplification of the resulting infinite series. Q.E.D.

**PROPOSITION 3.** Assume that prices are set by (1) with current and past dividends given by (9) as information and that  $|1/(1 + r)| < 1.0$ .

Then

$$\begin{aligned} \text{var}\{p_t^* | \Phi_{t-k}\} &= \text{var}\{p_t^* | p_{t-k}\} \\ &= \sigma_\eta^2 a^2 \left[ \left( \frac{1 - \rho^{2k}}{1 - \rho^2} \right) + \frac{(1+r)^2}{\rho^2(2r+r^2)} \right] \\ &= \text{var}\{p_t | p_{t-k}\} + \frac{\sigma_\eta^2 a^2 (1+r)^2}{\rho^2(2r+r^2)}. \end{aligned} \tag{12}$$

*Proof.* Follows from the definition (2) and the dividend process (9), conditioning on  $d_{t-k}$ , and simplifying the resulting infinite series. Q.E.D.

Note that in (12) the difference between the conditional variances  $\text{var}\{p_t^* | \Phi_{t-k}\}$  and  $\text{var}\{p_t | \Phi_{t-k}\}$ , which by (8) equals  $\text{var}\{\xi_t | \Phi_{t-k}\}$ , is for this case a constant equal to  $\text{var}\{p_t^* | \Phi_t\}$ . Note also that the restriction on  $r$  in proposition 3 prohibits  $-2 \leq r \leq 0$ , which ensures that the denominator in the expression for  $\text{var}\{p_t^* | \Phi_t\}$  in (12) is positive.<sup>5</sup>

It can be verified that the limits (as  $k \rightarrow \infty$ ) of the conditional variances in (11) and (12) equal the corresponding unconditional variances:

$$\text{var}(p_t) = \frac{\sigma_\eta^2 a^2}{1 - \rho^2}, \tag{13}$$

$$\text{var}(p_t^*) = \frac{\sigma_\eta^2 (1+r+\rho)}{(1+r-\rho)(1-\rho^2)(2r+r^2)}. \tag{14}$$

Further, for the random walk case ( $\rho = 1.0$ ), we have

$$\lim_{\rho \rightarrow 1} \text{var}\{p_t | p_{t-k}\} = \frac{\sigma_\eta^2 k}{r^2} \tag{15}$$

and

$$\lim_{\rho \rightarrow 1} \text{var}\{p_t^* | p_{t-k}\} = \frac{\sigma_\eta^2}{r^2} \left[ k + \frac{(1+r)^2}{2r+r^2} \right]. \tag{16}$$

This shows that the unconditional variances of  $p_t$  and  $p_t^*$  are not defined for the random walk, so that strictly speaking the bound (4) involves undefined terms. However, the corresponding variances satisfy inequality (8).

Throughout this section, the interpretation of the variances has been in the cross-sectional sense of (unobserved) variances at  $t$  across different possible economies. To illustrate this notion, we now show

<sup>5</sup> See Kleidon (1986) for an interpretation of this condition in terms of the time-series process for  $p_t^*$ .

the values for  $p_1$  and  $p_1^*$  for 20 replications of the simulated economy used to generate figure 2. The model used is the (geometric) random walk for prices traditionally used in finance, and it is shown in Section III below that this model is consistent with Standard and Poor's prices used in figures 1 and 3. The dividend process is<sup>6</sup>

$$\ln d_t = \mu + \ln d_{t-1} + \epsilon_t, \quad (17)$$

where  $\epsilon_t$  is i.i.d.  $N(0, \sigma^2)$ . We then have the following proposition.

**PROPOSITION 4.** Assume that prices are generated by (1) with current and past dividends given by (17) as information. Then the implied price is

$$p_t = \left( \frac{1+g}{r-g} \right) d_t, \quad (18)$$

where  $1+g \equiv \exp[\mu + (\sigma^2/2)]$ .

*Proof.* From (17), the lognormality of  $\exp(\mu + \epsilon_t)$  and the standard result for its expectation, and the independence of  $\epsilon_t, \epsilon_\tau, \tau \neq t$ , we have

$$E\{d_{t+\tau}|d_t\} = d_t(1+g)^\tau,$$

where  $g$  is defined in (18). Substitution into (1) gives (18) directly. Q.E.D.

Figure 4 shows price and  $p^*$  at the same date  $t = 1$  across 20 economies that were identical at  $t = 0$  but are different at  $t = 1$ . In each economy the starting price is set as  $p_0 = 40.0$ , and the same dividend process given by (17) is used in each replication—all that change are the random innovations  $\epsilon_t$ .<sup>7</sup> The first seed chosen arbitrarily for the random number generator produces the observations for "economy 1" used for figure 2, and subsequent seeds are produced internally by the IMSL generator.

From (8), we know that the variance of  $p_1$  given  $p_0$  should be less than the variance of  $p_1^*$  given  $p_0$ , and figure 4 shows precisely this result. Values of  $p_1$  vary across the 20 economies from a low of 30.48 for economy 10 to a high of 61.35 for economy 17. Much greater variability across economies is seen in  $p_1^*$ , as the theory predicts, and values range from 8.99 (economy 4) to 477.83 (economy 6).

To complete the picture, figure 5 shows *time-series* plots of 100

<sup>6</sup> No dividend smoothing is assumed, which is conservative since Standard and Poor's dividend series since about 1950 appears much smoother than either prices or (accounting) earnings. Section III discusses the implications of dividend smoothing in more detail.

<sup>7</sup> The values for the drift  $\mu$  and the innovation variance  $\sigma^2$  are estimated from first differences of logs of Standard and Poor's (real) price series for 1926–79, and Standard and Poor's (real) price index in 1926 is approximately 40.0. The series  $\epsilon_t$  are generated using the IMSL subroutine GGNPM. For more details, see Kleidon (1983).

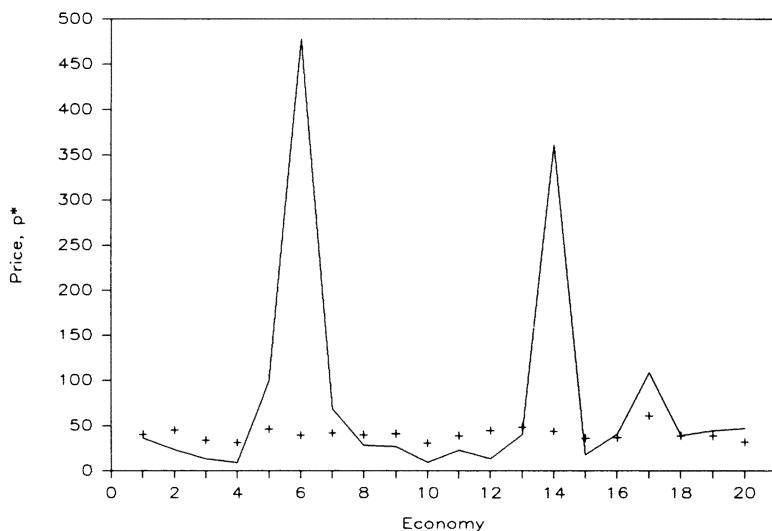


FIG. 4.—Distribution of  $p^*$  (solid line) and prices at time 1 across 20 economies that are identical at time 0. Note that this is not a time-series plot for one economy but the values at time  $t = 1$  across 20 different economies.

observations of  $p_t$  and  $p^*$  for three of the 20 economies shown in figure 4.<sup>8</sup> The three economies are 2, 4, and 6; the latter two are chosen because they give the lowest and highest values of  $p^*$ , respectively. It is obvious from figure 5 that the wide variation in  $p^*$  is simply the result of different ex post draws of dividends over time for the different economies. Each is possible at time 0 since the same stochastic process and same initial price  $p_0$  prevail in each economy. Ex post, quite different worlds could be encountered, and each implies its own value of  $p^*$ . The variance bounds hold across these different economies.

Although figures 4 and 5 show clearly the notion underlying variance bounds tests, the luxury of observing different worlds that may unfold through time is limited to theory or simulation. In reality we observe only one world. I now consider the properties one should expect to find in time-series plots for one economy.

## 2. Resolution of the Apparent Paradox

The current consensus has interpreted “smoothness” or lack of “short-term variation” in  $p^*$  relative to price as evidence against the

<sup>8</sup> The first economy is shown in fig. 2, and plots for the first 10 economies are given in Kleidon (1983, app. A).

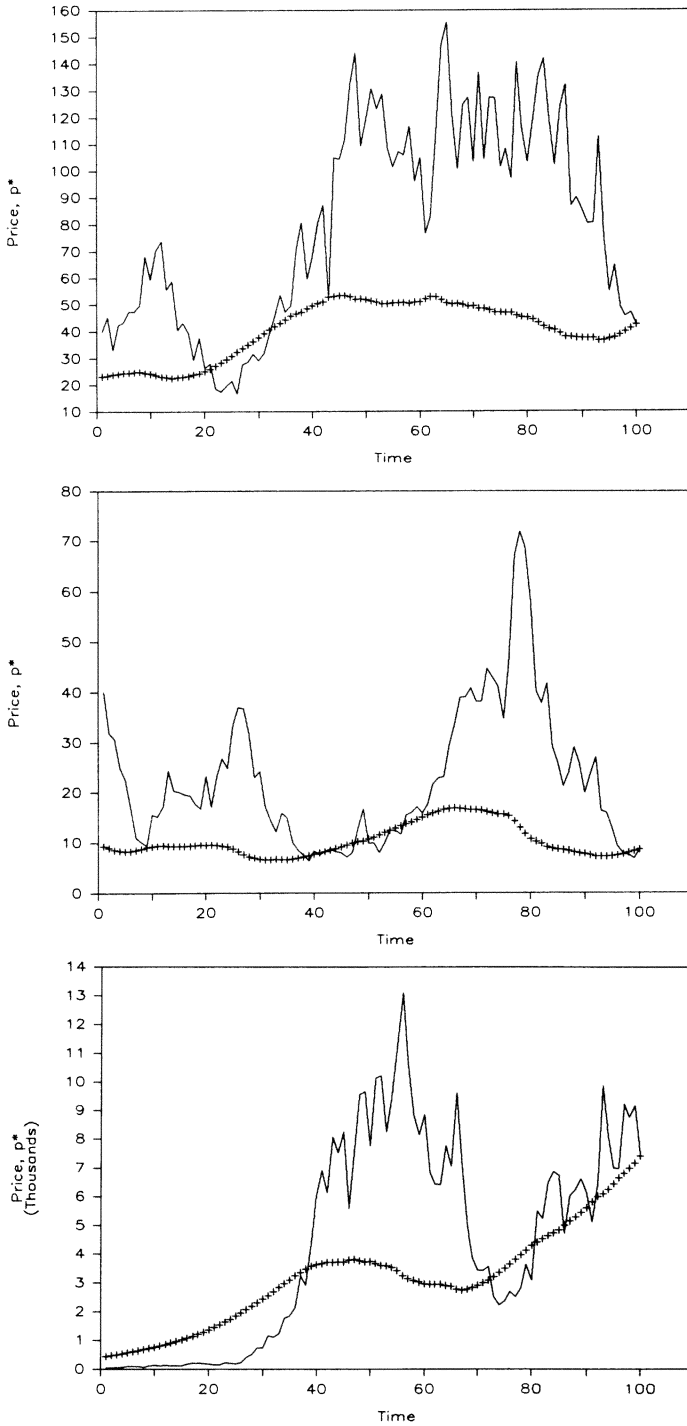


FIG. 5.—Plots of time series of nonstationary price series (solid line) and corresponding  $p^*$  series, for economies 2, 4, and 6 from 20 replications shown in fig. 4.

inequality (4). Although the terms are not explicitly defined in the literature, it seems reasonable to interpret the comments about smoothness or short-term variation as relating to the conditional variance of the series, given past values of that series. Thus I interpret the smoothness of price and  $p_t^*$  to be determined by  $\text{var}\{p_t|p_{t-k}\}$  and  $\text{var}\{p_t^*|p_{t-k}^*\}$ , respectively. Lack of short-term variation in  $p_t^*$  versus  $p_t$ , which led Grossman and Shiller (1981) to reject the valuation model (1), is consequently defined here to mean that, for small  $k$ ,

$$\text{var}\{p_t^*|p_{t-k}^*\} < \text{var}\{p_t|p_{t-k}\}. \tag{19}$$

Since the issue concerns conditional variances, it is natural to examine the general bound (8), which is written in terms of conditional variances.<sup>9</sup> It is immediately apparent that the conditional distribution  $\{p_t^*|p_{t-k}^*\}$  does not appear in (8)! Given the cross-sectional nature of these bounds, it could not since the variable  $p_t^*$  by (2) uses future dividend realizations that are not known at  $t$  and hence cannot be used as part of conditioning information at  $t$  to derive a valid bound. Consequently, despite the numerous references in the literature to the relative smoothness of price and  $p_t^*$ , this is a red herring with respect to variance inequalities.

It is clear that as  $k \rightarrow \infty$  the conditional distribution  $\{p_t^*|p_{t-k}^*\}$  approaches the unconditional distribution of  $p_t^*$ , so that the bound (4) will indeed hold for sufficiently large  $k$  (assuming the variances of the unconditional distributions exist). What is not obvious is the behavior of  $\{p_t^*|p_{t-k}^*\}$  for  $k$  small. We now show exactly what happens to the three conditional variances that appear in (8) and (19) as  $k$  changes for the dividend model (9). We have already seen that, consistent with (8),  $\text{var}\{p_t|p_{t-k}\} < \text{var}\{p_t^*|p_{t-k}\}$ . It remains to show the relation between  $\text{var}\{p_t|p_{t-k}\}$ , which determines the smoothness of prices, and  $\text{var}\{p_t^*|p_{t-k}^*\}$ , which determines the smoothness of  $p_t^*$ .

**PROPOSITION 5.** Assume that prices are set by (1) with current and past dividends given by (9) as information, that  $\eta_t$  is normally distributed, and that  $|1/(1 + r)| < 1.0$ . Then

$$\text{var}\{p_t^*|p_{t-k}^*\} = \text{var}(p_t^*)(1 - \rho_k^2), \tag{20}$$

<sup>9</sup> An earlier version of this paper distinguished between conditional variances similar to (19) and the unconditional variances in (4), and this argument is adopted in LeRoy (1984) using the conditional variances in (19). The current comparison of the conditional variances in (8) with those in (19) has the advantage of showing that the problem is not primarily with the use of conditional vs. unconditional variances, but with the use of incorrect conditional variances in (19).



where

$$\begin{aligned} \rho_k &\equiv \frac{\text{cov}(p_t^*, p_{t-k}^*)}{\text{var}(p_t^*)} \\ &= \frac{\rho^{k+1}(2r + r^2) - (1 - \rho^2)(1 + r)^{1-k}}{(1 + r + \rho)(\rho + r\rho - 1)}. \end{aligned}$$

*Proof.* Equation (20) follows directly from the normality of  $p_t^*$ , the definition of  $\text{var}(p_t^*)$  is given in (14) above, and  $\text{cov}(p_t^*, p_{t-k}^*)$  is straightforward to calculate given the definition (2) and the dividend process (9). Q.E.D.

It can be verified that the limit (as  $k \rightarrow \infty$ ) of  $\text{var}\{p_t^* | p_{t-k}^*\}$  in (20) is  $\text{var}(p_t^*)$  and that for the random walk case ( $\rho = 1.0$ )

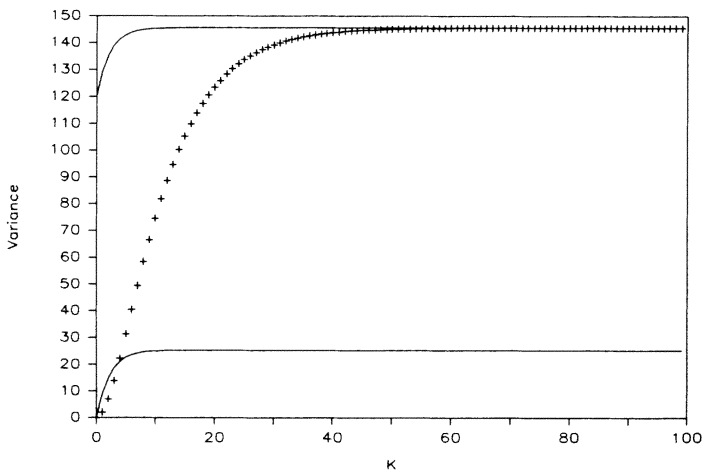
$$\begin{aligned} \lim_{\rho \rightarrow 1} \text{var}\{p_t^* | p_{t-k}^*\} &= \\ &= \frac{\sigma_\eta^2[(k + 1)(2r + r^2) - (1 + r)(3 + r) + 2(1 + r)^{1-k} + 1]}{r^2(2r + r^2)}. \end{aligned} \quad (21)$$

Again in this case, the conditional variances  $\text{var}\{p_t^* | p_{t-k}^*\}$  are well defined for  $k < \infty$ .

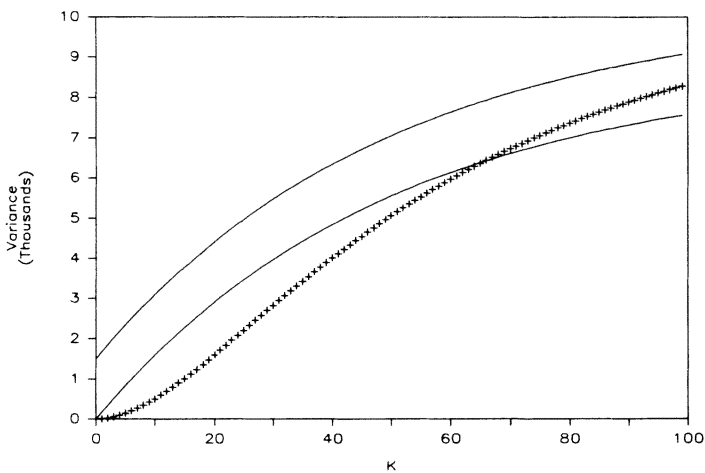
Figure 6 shows the relevant conditional variances  $\text{var}\{p_t | p_{t-k}\}$ ,  $\text{var}\{p_t^* | p_{t-k}\}$ , and  $\text{var}\{p_t^* | p_{t-k}^*\}$ , assuming  $r = 0.065$  and  $\sigma_\eta^2 = 1$ . Parts *a*, *b*, and *c* each show the three conditional variances for  $k$  from 0 to 100, for values of  $\rho = 0.80, 0.99,$  and  $1.0$ , respectively. As  $k$  increases, both  $\text{var}\{p_t | p_{t-k}\}$  and  $\text{var}\{p_t^* | p_{t-k}\}$  increase, and by (12) the difference is the constant  $\text{var}\{p_t^* | p_t\}$ . The inequalities in (8) are never violated, although for the random walk in part *c* of figure 6 both variances increase without bound.

Particularly interesting is the behavior of  $\text{var}\{p_t^* | p_{t-k}^*\}$  relative to  $\text{var}\{p_t | p_{t-k}\}$ , which determines the relative smoothness of the series. Both equal zero at  $k = 0$ , and for some value  $k$  (which increases in  $\rho$ ) it must be the case that  $\text{var}\{p_t^* | p_{t-k}^*\} > \text{var}\{p_t | p_{t-k}\}$  since we know that eventually the unconditional variances of  $p_t^*$  and  $p_t$  satisfy this inequality (assuming they exist). The key result, however, is that short-term variances show the opposite result, just as noted by Grossman and Shiller (1981). For  $k$  small, we see that  $\text{var}\{p_t^* | p_{t-k}^*\} < \text{var}\{p_t | p_{t-k}\}$ , and this can hold for quite large  $k$  depending on the parameter  $\rho$  in the dividend process.

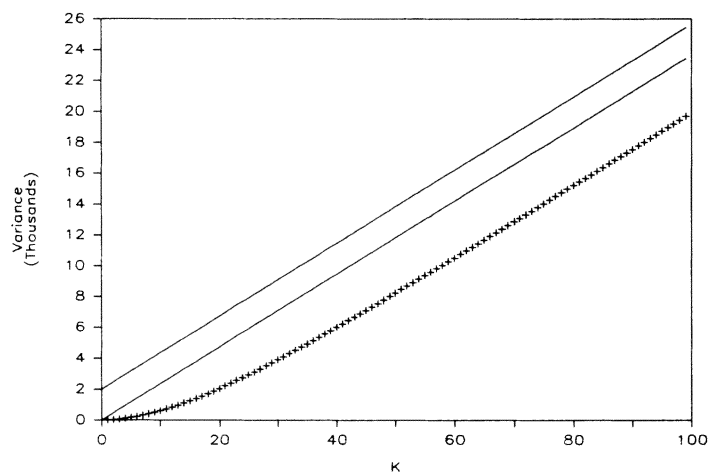
This implies that plots such as figures 1 and 3 above *should* show greater smoothness in  $p_t^*$  than in the price series if prices are given by (1). Such smoothness provides no evidence against either the bound (4) or the valuation model (1) but, on the contrary, is to be expected. Consequently the evidence used by Grossman and Shiller (1981) to



(a)



(b)



(c)

FIG. 6.—Conditional variances  $\text{var}\{p_t^*|p_{t-k}\}$  (upper solid line),  $\text{var}\{p_t|p_{t-k}\}$  (lower solid line), and  $\text{var}\{p_t^*|p_{t-k}^*\}$  (broken line),  $k = 0, \dots, 100$ , for AR(1) prices and dividends with (a)  $\rho = 0.80$ , (b)  $\rho = 0.99$ , and (c)  $\rho = 1.0$ .

conclude that prices cannot be given by (1) does not support their conclusion.

The intuition behind this result is straightforward. The series  $p_t^*$  is constructed so that ex post the sum of dividend yield and capital gain always gives exactly the rate  $r$  by (2). Consequently, changes in  $p_t^*$  will by construction give just the capital gain, which, together with the dividend  $d_t$ , ensures the total return  $r$ . Prices, however, can and frequently will show short-term changes of an order of magnitude larger than this since changes in current dividends in general imply changes in expected dividends for the infinite future. The price will change by the present value of these revisions in expected future dividends. Since by assumption the series  $p_t^*$  is already calculated using the ex post infinite dividend series, changes in current dividends imply no new information and no unexpected changes in  $p_t^*$ .

Given an understanding of what should be expected in time-series plots of price and  $p_t^*$ , we turn now to the issue of correspondence between the series.

## B. Correspondence between $p_t^*$ and $p_t$

### 1. The Argument

Grossman and Shiller (1981) rely on the relative degree of correspondence between two  $p_t^*$  series (with constant and nonconstant rates) and the price series  $p_t$  to determine which model is preferable, and they argue (p. 224) that “the rough correspondence between [ $p_t^*$ ,  $A = 4$ ] and [ $p_t$ ] (except for the recent data) shows that if we accept a coefficient of relative risk aversion of 4, we can to some extent reconcile the behavior of [ $p_t$ ] with economic theory *even under the assumption that future price movements are known with certainty*” (emphasis added).

The statement concerning a certainty assumption is crucial, and we return to it shortly. A more recent claim that the price series should correspond to the  $p_t^*$  series is one of the strongest. Shiller (1984) relies exclusively on plots such as figure 3 as a “particularly striking way of presenting the evidence” that stock price changes cannot be explained in terms of “some new information about future earnings” (p. 30). He uses virtually the same plot as figure 3 (extended to 1981) and claims (p. 31):

[Figure 3] shows that actual dividend movements of the magnitude “forecast” by price movements never appeared in nearly a century of data. We *might* have observed big movements in [ $p_t^*$ ,  $A = 0$ ] that correspond to big movements in [ $p_t$ ] and that would mean that movements in [ $p_t$ ] really did appropriately forecast movements in future dividends. On

the other hand, this just did not happen. Look, for example, at the stock market decline of the Great Depression, from 1929 to 1932.  $[p_t^*, A = 0]$  did go down then, but only very slightly, far less than the decline in  $[p_t]$ . The reason is that real dividends declined substantially only for the few worst years of the Depression. These few lean years have little impact on  $[p_t^*, A = 0]$ , which depends in effect on the longer-run outlook for stocks.

## 2. Analysis

Section IIA demonstrates that, even if cross-sectional variance bounds are satisfied, time-series plots of price and  $p_t^*$  will frequently show the series  $p_t^*$  as being much smoother than the price series if there is uncertainty about future dividends when prices are set. Consequently, it is not surprising that the series do not correspond to each other. What is crucial is how much information is available, which determines the degree of correspondence that should be expected. It is clear from the simulations in figures 2 and 5 that the amount of uncertainty about future cash flows implicit in the traditional geometric random walk is sufficient to imply the degree of divergence between  $p_t^*$  and  $p_t$  shown in Standard and Poor's series in figure 1.

Shiller's (1984) argument that the stock price should not have declined as much as it did between 1929 and 1932 because dividends declined substantially only in the few worst years of the depression assumes that stockholders knew that the lower dividends they were seeing would not last far into the future. Grossman and Shiller (1981) are more explicit and add an assumption of certainty about future prices. This assumption is not part of the model ostensibly being tested. The original model, given as (1) above, writes price in terms of expected future dividends, in contrast to  $p_t^*$ , which uses the ex post outcomes. In a world of certainty we would expect  $p_t^*$  to correspond to the actual price series—if discount rates were estimated correctly and the price series were rational, they should be identical.

But of course the actual stock prices shown in figure 3 were not set in a world of omniscience. If Grossman and Shiller's  $p_t^*$  series with nonconstant discount rates exactly corresponded to the actual price series, it would be misleading to claim that the series were consistent with economic theory "even under the assumption that future prices are known with certainty." Rather, there would be consistency with economic theory *only* under certainty since the price series will follow the ex post series exactly only if shareholders have perfect information about the future dividend series. If they do not—which is surely

the state of things—then one should expect deviation between *ex ante* and *ex post* prices.

The question then is not whether the  $p_t$  and  $p_t^*$  series deviate, but rather how much they deviate. It is initially tempting to regard the  $p_t^*$ ,  $A = 4$  series as preferable to the  $A = 0$  series because it more closely resembles actual prices  $p_t$ . But until we specify how much the  $p_t^*$  series *should* deviate from the price series—that is, until we specify the amount of uncertainty in the market about future cash flows—we cannot decide which plot deviates by the correct amount. The issue is addressed by Shiller (1984, p. 35), but he does not present sufficient evidence to allow inference about the degree of divergence to be expected: “Of course, people do not have perfect foresight, and so actual stock prices [ $p_t$ ] need not equal [ $p_t^*$ ]. We [i.e., Grossman and Shiller (1981)] argue that even under imperfect information we might expect [ $p_t$ ] to resemble [ $p_t^*$ ], though if information is very bad the resemblance could be very weak.” This illustrates precisely the difficulty in examining plots such as figures 1 and 3. Until we know how imperfect the information is, we cannot interpret how weak the resemblance should be. A fundamental misinterpretation of such figures has been to make inferences about the validity of the valuation model (1) without specifying the yardstick necessary to allow such inferences.

To see whether the degree of correspondence between  $p_t^*$  and price in figure 1 is consistent with the valuation equation (1), we need a model that specifies the information available to the market about future cash flows. One possibility—favored by Grossman and Shiller—is to assume that shareholders have a large amount of information about future dividends. Then the only way prices could be rational is if discount rates vary greatly because of changes in aggregate consumption, which is their solution. Unfortunately, as discussed in more detail below, this solution fails when applied to other data.

An alternative explanation is much more consistent with the data. Using the (geometric) random walk for prices traditionally used in finance and assuming that the only information available at time  $t$  is the past history of dividends, we see in figures 2 and 5 that there is sufficient uncertainty about future cash flows to imply the large divergence between prices and  $p_t^*$  seen in Standard and Poor’s data in figures 1 and 3. The procedures used to construct figures 2 and 5 are conservative since discount rates are strictly constant by construction and no dividend smoothing is assumed.<sup>10</sup>

<sup>10</sup> Hence Marsh and Merton (1984a, p. 19) are incorrect in claiming that “[fig. 3] can be interpreted as implying that the  $p_t^*$  series has ‘too little’ volatility to be consistent with a dividend process which is not smoothed.”

### C. Conclusion

This section has demonstrated that plots such as figures 1 and 3 cannot be regarded as inconsistent with the valuation model (1), although at first they appear to be convincing evidence against its validity. It is tempting to look at the  $p_t^*$  series as the “true” price, which does not vary much through time, and the actual price as (correlated) deviations from the true price. Such an interpretation is incorrect because the price at  $t$  can only be assessed relative to the information  $\Phi_t$ . Thus in figure 2 the actual price series is by construction the conditional expectation of  $p_t^*$  given  $\Phi_t$ , and by construction the prediction error  $\xi_t$  in (7) (i.e., the difference between this conditional expectation and the ex post outcome for  $p_t^*$ ) is uncorrelated with  $p_t$  or with past prices, which are also in  $\Phi_t$ .

What is potentially misleading from figure 2 is that successive prediction errors are highly correlated with each other, which appears to contradict the previous statement. Again, however, the problem lies in the information that is implicitly being used for conditioning. Previous forecast errors  $\xi_{t-k}$  are not in the information set at  $t$  since previous  $p_{t-k}^*$  that depend on the ex post outcomes for future dividends are unknown at  $t$ . Clearly the errors will be correlated since almost the same future set of dividends are being forecast at, say,  $t$  and  $t + 1$ .<sup>11</sup> As seen in figures 4 and 5, the errors across economies at time  $t$  are indeed unrelated to prices at  $t$ .

Despite the potential for confusion in plots such as figure 3, they have been heavily relied on in the literature and have even been treated as stronger evidence against (1) than formal tests of the bound (4). Shiller (1981*a*, pp. 4, 7; 1984) claims that figure 3 alone is sufficient to show that stock prices are inconsistent with the valuation model (1), as does Tirole (1985). This is simply incorrect. However (as Shiller [1981*a*] points out), the more formal tests of (4) based on time-series data for a single economy are also problematic, and I now turn to them.

### III. Time-Series Tests of Variance Bounds

The assumption typically made to test the bound (4) using time-series data is that the relevant variables (namely, dividends and prices for the dividend discount model being discussed here) follow stationary and ergodic processes. If this is true, then the sample moments are consistent estimators for the moments of the unobservable distribu-

<sup>11</sup> The issue of overlapping forecast errors also arises in other contexts, e.g., spot and forward foreign exchange rates (see Hansen and Hodrick 1980).

tions used in the inequality, assuming a sufficiently long time series of realizations from those distributions.<sup>12</sup>

Shiller (1981*b*, 1981*c*) tests the bound (4) with Standard and Poor's and Dow Jones Industrial Average indexes of annual stock prices and dividends (1981*b*, pp. 434–35), using sample variances of price and  $p_t^*$  as estimators of unconditional population variances. He reports that the bound appears grossly violated but does not conduct formal significance tests. LeRoy and Porter (1981) also test (4) but derive it from Miller and Modigliani's (1961) model based on future earnings  $X_t$  and investments  $I_t$ :

$$p_t = \sum_{\tau=1}^{\infty} \frac{E\{(X_{t+\tau} - I_{t+\tau})/n_t | \Phi_t\}}{(1+r)^\tau}, \quad (22)$$

where  $r$  is an assumed constant discount rate and  $n_t$  is the number of shares outstanding at  $t$ .<sup>13</sup> LeRoy and Porter conduct formal tests of the bound under the assumption of stationarity of their series. The point estimates imply violation for Standard and Poor's data, but sampling error is sufficiently high that the bound is not rejected at conventional significance levels (p. 557). Tests on individual stocks indicate rejection.

However, there are at least two important reasons to question whether the extra assumptions underlying these tests are valid empirically. First, as documented in Section IIIC below, the data used in many of the variance bounds tests are consistent with the assumption that prices follow a nonstationary random walk. If so, the unconditional variances in (4) do not exist, and the use of sample variances of  $p_t$  and  $p_t^*$  as estimators of population unconditional variances is invalid. Section IIIA shows that the apparent gross violations of the variance bound (4) reported in the current literature using sample variances of  $p_t$  and  $p_t^*$  are consistent with an incorrect assumption of stationarity of prices and dividends. However, it is valid to estimate conditional variances even if prices are nonstationary, and Section

<sup>12</sup> See Fuller (1976, p. 230). Just how long is "sufficient" in this context, even assuming stationarity and ergodicity, is investigated in detail in Kleidon (1983, chap. 5; 1986). See also Flavin (1983).

<sup>13</sup> They do not use exactly this model but use  $n_{t+\tau}$  as the divisor, which implicitly assumes that the net benefits of future investments do not accrue to current shareholders. In private correspondence, Stephen LeRoy indicates that this adjustment makes little difference. Two other issues are of greater potential significance. First, LeRoy and Porter (1979, pp. 2, 3) adjust prices and earnings to account for earnings retention. Although this is feasible under certain conditions, their procedure uses an incorrect timing assumption that violates the dividend irrelevance proposition. Second, their results are based on incorrect data since in effect they create an artificial Standard and Poor's price series with a spurious seasonal at lag 4, as shown in their table 4 (1981, p. 570). For details, see Kleidon (1983, chap. 3).

IIIB shows that Standard and Poor's data do not violate the conditional variance inequalities in (8).

LeRoy and Porter discuss the assumption of stationarity of earnings and prices in some detail and make adjustments for earnings retention. Shiller (1981c, p. 293) claims that "the resulting series appear stationary," but LeRoy and Porter report that, after their adjustments, there remains evidence of nonstationarity.<sup>14</sup> They continue (1981, p. 569): "We have decided to neglect such evidence and simply assume that the series are stationary. . . . We do not argue that this treatment is entirely adequate, nor do we in any way minimize the problem of nonstationarity; the dependence of our results on the assumption of stationarity is probably their single most severe limitation."

The second problem, that of dividend smoothing, has important implications for all research that attempts to infer the properties of an infinite stream of future dividends from some finite ex post set of dividends that are under some control of management. Empirical evidence suggests that management takes care to create a smooth short-run dividend series that may not reflect one for one the fortunes of the firm as determined primarily by its earnings and investment opportunities.<sup>15</sup> *Ceteris paribus*, the less variable the dividend stream, the more variable will be the price series that comprises the present value of future dividends. For example, a firm seeking to finance expansion internally may withhold all dividends over some finite period, with an implicit promise of some future (perhaps liquidating) dividend.<sup>16</sup>

If dividends are smoothed, the time series may be covariance nonstationary and violate the assumption of ergodicity necessary to allow estimation of valid cross-sectional variance bounds with time-series data. To illustrate, suppose that at  $t$  there exists a firm that has future cash flows per share  $D$  composed of earnings (paid out fully as dividends) at only one period, say  $T$ , and suppose for convenience that the discount rate  $r = 0$ .<sup>17</sup> This implies that the conditional distribution  $\{p_t^* | \Phi_t\}$  is just the conditional distribution  $\{D | \Phi_t\}$ . Clearly the bound (4) holds at  $t$  assuming that the relevant variances are defined.

<sup>14</sup> Since LeRoy and Porter attempt to correct for nonstationarity, the results in this section based on original data apply to their work only to the extent that nonstationarity remains.

<sup>15</sup> See, e.g., Lintner (1956) and Fama and Blasiak (1968). This does not deny that dividends may contain some information, as in the signaling hypotheses of Ross (1977) and Bhattacharya (1980).

<sup>16</sup> Note that Marsh and Merton's (1984a) definition of dividend smoothing does not deal with this case since it does not allow firms to pay zero dividends in any period when the price is positive (see their eq. [7], p. 13).

<sup>17</sup> Paul Pflleiderer suggested this example, say for the case of a firm drilling for oil.



However, the ex post time series  $p_t^*$ , calculated from the recursion (5) and based on the terminal payment, will be a constant with zero sample variance. The price series will show positive variance if information about the terminal payment becomes available through time so that the bound (4) will appear violated if estimated by sample variances.

The problem is more severe for inequalities that, unlike (4) or (8), are invalid if an assumption of ergodicity of dividends is violated. The variance inequality that has received most attention in the literature is (4), but others exist. Some, such as LeRoy and Porter's bound (1981, p. 560) on the coefficient of dispersion (i.e., the ratio of the standard deviation to the mean), are similar to (4) in that they rely on stationarity and ergodicity assumptions for testing, but not for the intrinsic validity of the bound. Others such as in Shiller (1981c) are based on the time series of prices and dividends, and so rely on some form of stationarity for their validity, even aside from issues of testing. Shiller's alternate bounds are given as (1981c, p. 296, eqq. I-2, I-3)

$$\sigma(\Delta p) \leq \frac{\sigma(d)}{\sqrt{2r}}, \quad (23)$$

$$\sigma(\Delta p) \leq \frac{\sigma(\Delta d)}{\sqrt{2r^3/(1+2r)}}, \quad (24)$$

where  $\sigma(\cdot)$  is standard deviation,  $\Delta p$  and  $\Delta d$  are first differences of price and dividends, respectively, and  $r$  is the (assumed constant) one-period discount rate. The derivation of (23) assumes joint covariance stationarity of the time series  $p_t$  and  $d_t$ , while that of (24) assumes joint stationarity of  $\Delta p_t$  and  $\Delta d_t$ , with information variables contained in the information set (Shiller 1981c, pp. 295–97). Only (24) is consistent with nonstationary (random walk) prices and dividends, and only (24) is not violated by point estimates.<sup>18</sup> However, the assumptions underlying both (23) and (24) may be violated if dividends are smoothed.

The issue of dividend smoothing can have striking implications for some more recent tests that attempt to overcome criticisms of early variance bounds tests. For example, West (1984) derives and tests the inequality that the variance of changes through time in the present value of expected dividends will be greater when the information set comprises current and past dividends than when it comprises a larger set. Although he regards the necessary assumption that dividends

<sup>18</sup> See Shiller (1981c, p. 297). He continues: "Of course, we do not expect the data to violate all inequalities even if the model is wrong" and notes that, although this inequality is not violated for first differences of the data, the relevant bound is violated when the data are differenced using an interval of 10 years (i.e.,  $x_t - x_{t-10}$ ). Section III B discusses this claim in the context of comparable results based on conditional variances.

follow an autoregressive integrated moving average (ARIMA) process as “relatively mild” (p. 3), this can be violated if dividend smoothing implies changes in a future residual dividend that do not show up in the currently observed dividend series. In the extreme, if the finite and observed dividend series were constant, the use of only this stream to predict future dividends would imply constant future dividends, and so the present value would be constant through time and the innovation zero. In West’s terminology, this would appear to be evidence that 100 percent of price changes could be attributed to speculative bubbles—in fact, the violation of the assumption of an ARIMA process for dividends simply means that the theoretical inequality is invalid.<sup>19</sup>

Although the issue of dividend smoothing is potentially very important in interpreting the results from any particular test, the remainder of this section assumes nonsmoothed dividends as in figures 2 and 5 to highlight the implications of nonstationarity, which is most crucial in the current context. First, Section IIIA discusses the nonstationary price model used in figure 2, derives consistent dividend and earnings models, and shows that the current gross violations of the bound (4) are not surprising if prices follow this process with parameters corresponding to Standard and Poor’s price data. Section IIIB derives conditional variance inequalities that are valid for the nonstationary price process and demonstrates that Standard and Poor’s series do not violate these bounds. Section IIIC completes the argument by showing empirically that the assumed process is consistent with Standard and Poor’s data.

#### A. *Nonstationary Prices and Tests of Unconditional Variances*

Stationarity of stock prices is vital to the validity of much of the variance bounds literature. The cited tradition in finance for treating stock prices as nonstationary random walks goes back to at least 1934 when it was recognized “that stock prices resemble cumulations of purely random changes” (Working [1934]; cited in Roberts [1959, p. 2]). Annual accounting earnings also appear to be well described as

<sup>19</sup> Similar issues arise in recent attempts to account for nonconstant discount rates in (1). For example, Scott (1984) uses Hansen’s (1982) generalized method of moments estimator and assumes in one specification that dividends are not mean-reverting to avoid criticisms concerning assumed stationarity of dividends. However, in this case he assumes (p. 8) that “the percentage change in dividends and stock prices as well as the price-dividend ratio ( $\Delta D/D$ ,  $\Delta P/P$ ,  $P/D$ ) are stationary.” Such an assumption may be violated if dividends are smoothed.

random walks.<sup>20</sup> However, most variance bounds tests assume stationarity of stock prices, dividends, or earnings, usually after deflation by some price index to account for inflation, and “detrending” to remove a perceived deterministic time trend. Nelson and Plosser (1982) compare these two approaches and cannot reject that stock prices (as well as several other macroeconomic variables) are “non-stationary stochastic processes with no tendency to return to a trend line” (p. 139).<sup>21</sup>

The simplest random walk model, (10) above with  $\rho = 1.0$ , implies a zero expected capital gain component in stock returns, which historically is not true given less than full payout of earnings as dividends. An alternate model,

$$p_t = \mu + p_{t-1} + \epsilon_t, \quad (25)$$

where  $\epsilon_t$  is i.i.d. with mean zero and variance  $\sigma^2$ , implies an expected capital gain rate that varies inversely with the price level. The most plausible economic model in this context is a geometric random walk,

$$\ln p_t = \mu + \ln p_{t-1} + \epsilon_t, \quad (26)$$

or

$$p_t = p_{t-1} \exp(\mu + \epsilon_t). \quad (27)$$

If the capital gain rate is defined as  $(p_t - p_{t-1})/p_{t-1}$ , then the (conditional) expected capital gain rate ( $g$ ) is constant and is given by

$$\begin{aligned} g &\equiv E\left\{\frac{p_t - p_{t-1}}{p_{t-1}} \middle| p_{t-1}\right\} \\ &= \exp\left(\mu + \frac{\sigma^2}{2}\right) - 1.0, \end{aligned} \quad (28)$$

assuming lognormality of  $\exp(\mu + \epsilon_t)$  in (27). Expected capital gain rates are calculated below using (28).

### 1. Consistent Price, Dividend, and Earnings Processes

The valuation models based on dividends (1) and earnings or net cash flows (22) preclude any necessary one-to-one relation between the time-series process for price and the time-series process for earnings or dividends. As discussed with respect to dividend smoothing, any

<sup>20</sup> For empirical studies on annual earnings as a random walk, see, e.g., Little (1962), Ball and Watts (1972), Albrecht, Lookabill, and McKeown (1977), and Watts and Leftwich (1977). For quarterly earnings see Foster (1978).

<sup>21</sup> They note the implications of nonstationarity for variance bounds tests (pp. 142, 143), as do Black (1980) and Copeland (1983). See also Kling (1982).

particular set of observations may be unrepresentative of the total expected dividend stream. In principle, the same phenomenon could occur in the earnings stream (net cash flow) approach since the pattern of net cash flows does not necessarily correspond to changes in the present value of expected future net cash flows through time. Further, we do not observe the requisite “economic earnings” but accounting earnings. One cannot infer that a rational price series must be generated by a particular stochastic process just because dividends or earnings follow the process in a finite set of observations, or vice versa. However, one can infer a (nonunique) process for dividends or earnings that is compatible with the observed price series and see if the process is confirmed in dividend/earnings data.

This section assumes that prices follow the geometric random walk (26), defines consistent dividend and earnings processes, and discusses the underlying economic models. We have seen from proposition 4 above that one dividend process consistent with the price process (26) is

$$\ln d_t = \mu + \ln d_{t-1} + \epsilon_t, \tag{17}$$

where  $\mu$  and  $\epsilon_t$  are identical to those in (26), since

$$p_t = \left( \frac{1 + g}{r - g} \right) d_t. \tag{18}$$

Not surprisingly, one consistent earnings process also has constant expected growth and is analogous to the price (26) and dividend per share (17) processes. However, the earnings stream approach involves investment as well as earnings. To specify the expected growth rate in earnings and its relation to that in prices and dividends per share, two issues are important: Is investment financed internally (via retained earnings) or externally (via new capital issues), and how profitable are the investments?

**PROPOSITION 6.** If investment is a constant proportion  $\delta$  of earnings each period, is financed internally, and earns the rate of return  $r$ , then an earnings per share process consistent with the price process (26) and the dividend per share process (17) is

$$\ln e_t = \mu + \ln e_{t-1} + \epsilon_t, \tag{29}$$

and the relation between  $p_t$  and  $e_t$  is given by

$$\begin{aligned} p_t &= \frac{1}{r} E\{e_{t+1} | e_t\} \\ &= \frac{1 + g}{r} e_t, \end{aligned} \tag{30}$$

where  $e_t \equiv X_t/n_t$  and  $1 + g \equiv \exp[\mu + (\sigma^2/2)]$ .

*Proof.* Equation (29) follows from (26) if (30) holds, and the expected growth rate in earnings is  $g$ . But given the assumed investment process,  $g = \delta r$  (cf. Copeland and Weston 1983, p. 485). Further, since all financing is internal,  $d_t = (1 - \delta)e_t$ , and substitution into (18) gives (30).<sup>22</sup> Q.E.D.

Note that, although consistent processes for price and dividends were derived in terms of an unspecified empirical growth rate  $g$ , the earnings and investment model defines this rate in terms of fundamental variables,  $g = \delta r$ .

## 2. Tests of Unconditional Variances: Simulation Results

I now demonstrate that gross violations of bounds based on unconditional variances can result if the procedures of, say, Shiller (1981*b*) are applied to a series that by construction is rationally set by Miller-Modigliani valuation models with constant discount rates but that is nonstationary. This section reports the results of Monte Carlo simulations of the nonstationary price and dividend processes given above. The parameter values  $\mu = .0095$ ,  $\sigma = .218$ , and  $p_0 = 40.0$  are set to correspond to estimates for Standard and Poor's (deflated) annual price series, 1926–79. A series of disturbances  $\epsilon_t$  are generated by the IMSL subroutine GGNPM, and the dividend and corresponding rational price series are generated by (17) and (18). The  $p_t^*$  series is generated recursively by (5) with the terminal condition  $p_T^* = p_T$ .<sup>23</sup> The sample variances of the price series  $p_t$  and the  $p_t^*$  series are then calculated, and the variance bound (4) is deemed violated if for sample variances  $\text{var}(p_t) > \text{var}(p_t^*)$ . The procedure is carried out for two different but related price series. The first is the series constructed by (18), and the second “detrends” prices and dividends before calculating the corresponding  $p_t^*$  series following Shiller (1981*b*, p. 432).

Table 1 gives the percentage (across 100 replications) of violations of the variance bound (4) for the simulated price series (18) and its

<sup>22</sup> If external financing (from new securities) is raised for the investment, the growth rate in earnings will exceed  $g$ , the growth rate for prices and dividends per share (see Miller and Modigliani 1961, pp. 421–26). The assumption of normal returns on investment is less likely to be violated for the economy as a whole, as reflected in Standard and Poor's index, than for some particular “growth company.” If investment earns abnormal returns, compare Miller and Modigliani (1961, p. 423, eq. 25).

<sup>23</sup> Marsh and Merton (1984*a*) show that, if the terminal value  $p_T^*$  is set equal to the average sample price, the bound (4) is always violated if prices follow (26). This result does not hold for the terminal condition imposed by Grossman and Shiller (1981) and examined here (although Marsh and Merton [1984*b*, p. 12, n. 4] state the contrary). Their analysis does not show whether the “gross violations” of the bound that are reported in the literature can be explained by nonstationarity of prices, which is examined in table 2 below.

TABLE 1

PERCENTAGE OF VIOLATIONS OF VARIANCE BOUNDS CALCULATED FROM: (i) SIMULATED RATIONAL (GEOMETRIC) RANDOM WALK SERIES, (ii) THE SAME SERIES WITH EXPONENTIAL "DETRENDING," AND (iii) THE MATCHING CONSTRUCTED "PERFECT-FORESIGHT" PRICE SERIES

SAMPLE SIZE (T)	VIOLATIONS (%) OF VARIANCE BOUND: $\text{var}(p^*) \geq \text{var}(p)$						DURBIN-WATSON STATISTIC FOR RESIDUALS FROM DETRENDING IN (ii)	
	(i) Random Walk Series			(ii) "Detrended" Random Walk Series			Mean	Standard Deviation
	r = .05	r = .065	r = .075	r = .05	r = .065	r = .075		
5	100	100	100	95	95	95	2.37	.54
10	100	100	100	97	99	99	1.44	.52
50	83	83	88	89	90	90	.38	.19
100	86	87	87	90	92	92	.20	.10
200	86	87	88	92	92	93	.10	.05
1,000	95	95	95	100	100	100	.02	.01
2,000	91	91	91	96	96	96	.01	.006
3,000	72	71	71	99	99	99	.007	.005

NOTE.—Based on 100 replications. The relevant processes are: (i)  $\ln p_t = \mu_p + \ln p_{t-1} + \epsilon_t$ ,  $\epsilon_t$  i.i.d.  $N(0, \sigma^2)$ ,  $\ln d_t = \mu_d + \ln d_{t-1} + \epsilon_t$ ,  $\epsilon_t = ad_t$ , where  $a = (1 + \beta)/(v - \beta)$ ,  $(1 + \beta) = \exp[\mu_p + (\sigma^2/2)]$ , and  $r$  = a constant discount rate. Parameter values are  $\mu_p = 0.0095$ ,  $\sigma = 0.218$ , and  $\mu(0) = 40.0$ . (ii) The series are "detrended" (following Shiller [1981b, p. 482]):  $\hat{d}_t = d_t/e^{b(t-T)}$ ,  $\hat{p}_t = p_t/e^{b(t-T)}$ , where  $T$  is the base year, and  $b$  is estimated as the coefficient on the log of price on a regression on a constant and time. (iii) The "perfect-foresight" series corresponding to cases i and ii are defined as  $\hat{p}_t^* = (p_{t+1}^* + D_{t+1})/(1 + r)$ , where  $D_{t+1}$  = the dividend from  $t$  or  $t+1$ , and  $\hat{p}_t^* = pr$ , the terminal price in  $t$  or  $t+1$ .

detrended counterpart. Results are shown for three different (real) discount rates  $r$ , namely 0.05, 0.065, and 0.075. Over 1926–81, Ibbotson and Sinquefeld (1982, p. 15, exhibit 3) report an arithmetic mean nominal return per annum on the Center for Research in Security Prices (CRSP) file of common stocks of 0.114, with mean inflation of 0.031 per annum. Over the same period, the mean return for small stocks was 0.181, and Standard and Poor's index is composed of larger stocks. Shiller (1981*b*, p. 431, table 2) uses a discount rate of 0.048 per annum (in real terms) for detrended data or 0.063 per annum (p. 430) for nondetrended data. The rate 0.075 is given for comparison.

The most striking result of table 1 is the very high number of violations of the variance bound (4). The detrending procedure appears to exacerbate the tendency to reject the inequality, but the discount rates examined here do not appear to have much effect on the frequency of violation.<sup>24</sup> Table 1 also gives the mean and standard deviation (across replications) of the Durbin-Watson statistic from ordinary least squares (OLS) regression of prices on time, which is part of the detrending procedure. As noted below (n. 33), the average value for sample size 50 is almost identical to that obtained for the actual Standard and Poor's price series.

Although table 1 establishes that the variance bounds test procedures overwhelmingly result in violations of the bound (4) when applied to a nonstationary series generated by (1), it does not show whether gross violations are likely to occur. For 1,000 Monte Carlo replications for the sample size 100, which corresponds to Shiller (1981*b*, 1981*c*) and Grossman and Shiller (1981), table 2 gives both the number of violations of (4) (i.e., when the ratio of the sample standard deviation of price to the sample standard deviation of  $p^*$  exceeds 1.0) and the number of gross violations (when the ratio exceeds 5.0). Shiller (1981*b*, p. 341, table 5) reports a gross violation ratio of 5.59 for Standard and Poor's data. For rational, nonstationary series that are detrended, 397 replications out of 1,000 (or about 40 percent) give violation ratios greater than 5.0 using a discount rate  $r$  of 0.05, and 148 replications (almost 15 percent) for  $r = 0.065$ . Even for  $r = 0.075$ , almost 5 percent of the replications result in gross violations.

In short, the results of table 2 show that the gross violations of the bound (4) are not surprising if test procedures that assumed the existence of population unconditional variances were incorrectly applied

<sup>24</sup> As Joerding (1984) points out, the discount rate can in principle affect the frequency of violation. Further, table 2 shows that the discount rates examined here do affect the average amount by which the bound is violated.

TABLE 2

SUMMARY STATISTICS OF THE DISTRIBUTION OF THE RATIO OF SAMPLE STANDARD DEVIATION OF PRICE TO SAMPLE STANDARD DEVIATION OF "PERFECT-FORESIGHT" PRICE, ACROSS 1,000 REPLICATIONS FOR SAMPLE SIZE 100, FOR (i) SIMULATION RATIONAL (Geometric) RANDOM WALK SERIES AND (ii) THE SAME SERIES WITH EXPONENTIAL "DETRENDING."

	NUMBER OF VIOLATIONS (Ratio > 1.0)	NUMBER OF "GROSS VIOLATIONS" (Ratio > 5.0)	RATIO MEAN*	RATIO STANDARD DEVIATION	MINIMUM RATIO	MAXIMUM RATIO	PERCENTILE			
							50th (Median)	90th	95th	
Case i, random walk:										
$r = .05$	855	307	3.56	2.70	.51	15.59	2.62	7.22	8.40	
$r = .065$	865	62	2.65	1.50	.53	8.46	2.55	4.55	5.25	
$r = .075$	869	21	2.33	1.15	.57	7.48	2.39	3.75	4.31	
Case ii, "detrended" random walk:										
$r = .05$	894	397	4.29	3.04	.49	14.65	3.31	8.53	9.62	
$r = .065$	915	148	3.30	1.66	.51	8.86	3.42	5.39	6.09	
$r = .075$	925	46	2.90	1.24	.52	6.89	2.98	4.44	4.94	

NOTE.—The relevant processes are defined in table 1.

\* All summary statistics are for the distribution across 1,000 replications.



to nonstationary price data. Note that the parameter values used in these simulations are chosen as those estimated for Standard and Poor's price series, 1926–79, and that the nonstationary process (26) used here is consistent with the data. Note also that the simulations assume a dividend process with the same innovation variance as the price series, in (17) and (26). At least since 1950, Standard and Poor's dividend series is much smoother than the corresponding price or earnings series. Consequently, the simulation results are biased against finding gross violations relative to a dividend series with a lower sample innovation variance, and one would expect even greater rejection in actual empirical tests.

### B. Tests of Inequalities Based on Conditional Variances

This subsection tests the conditional variance inequalities given by (8), which are valid for nonstationary price series. The results show that the conditional bounds are not violated by Standard and Poor's data and provide both confirmation and interpretation of tests of inequality (24) above based on differences of prices and dividends.

We test variances of  $p_t$  and  $p_t^*$  conditional on past prices  $p_{t-k}$ , for  $k = 1, 2, 5, \text{ and } 10$ . The assumed price process is the geometric random walk (26), which implies that

$$\begin{aligned} \text{var}\{p_{t+k}|p_t\} &= \text{var}\{p_t \exp(\mu + \epsilon_{t+1})\exp(\mu + \epsilon_{t+2}) \dots \\ &\quad \exp(\mu + \epsilon_{t+k})|p_t\} \\ &= p_t^2 \text{var}\left[\exp\left(k\mu + \sum_{n=1}^k \epsilon_{t+n}\right)\right] \\ &\equiv p_t^2 c_k, \end{aligned}$$

where  $c_k$  is constant through time given  $\epsilon_t$  i.i.d.  $N(0, \sigma^2)$ .<sup>25</sup> Hence the conditional variances are constant through time except for scaling by  $p_t^2$ , and to avoid the resulting heteroscedasticity the inequality tested here is

$$\text{var}\left\{\frac{p_{t+k}}{p_t} \middle| p_t\right\} \leq \text{var}\left\{\frac{p_{t+k}^*}{p_t} \middle| p_t\right\}. \quad (31)$$

<sup>25</sup> As noted by Gary Chamberlain, more general distributional assumptions that allow conditional heteroscedasticity (i.e., nonconstant  $c_k$ ) are consistent with the tests conducted here, although in that case the bounds are in terms of the expected value of the conditional variances.

The population variances in (31) are constant through time for the price and dividend processes (26) and (17).<sup>26</sup> Note the equality of the conditional means,

$$\begin{aligned} E\left\{\frac{p_{t+k}}{p_t} \middle| p_t\right\} &= E\left\{\frac{p_{t+k}^*}{p_t} \middle| p_t\right\} \\ &= (1 + g)^k, \end{aligned} \quad (32)$$

where  $1 + g$  is the growth rate in prices as above. The conditional variances in (31) are estimated by the corresponding sample mean square deviations from the conditional means, using the sample estimated growth for the conditional means of  $p_{t+k}$  and  $p_{t+k}^*$  by (32).<sup>27</sup>

Table 3 reports the ratio of the conditional standard deviation of price to the conditional standard deviation of  $p_t^*$  for Standard and Poor's series, 1926–79, together with a sampling distribution for a sample size of 54, for discount rates of 0.05, 0.065, and 0.075. This distribution was constructed as in Section IIIA by simulation (here over 2,000 replications) of the price and dividend processes given by equations (26), (17), and (18), for parameter values corresponding to Standard and Poor's series.

There are two main results of interest in table 3. First, comparison of Standard and Poor's statistics with the corresponding sampling distribution shows that none of the inequalities given by (31) is violated at even a 10 percent significance level. Second, note that the point estimates do not violate (31) for  $k = 1, 2,$  or  $5$  but do violate for  $k = 10$ . It is significant that Shiller (1981c, p. 297) reports that the bound (24), which is consistent with nonstationary prices and dividends, is not violated when the data are differenced with a lag ( $k$ ) of 1 but are violated when  $k = 10$ . Although he treats this as an important rejection, he presents no formal significance tests. The simulation results here show that violation of the bound (31) by point estimates for  $k = 10$  is consistent with the valuation model (1).

Again, note that this sampling distribution is generated under the assumption of no dividend smoothing and, consequently, is conservative if dividends are smoothed. Even if smoothing is ignored, however, these tests show that Standard and Poor's price and dividend

<sup>26</sup> The use of a sample  $p_t^*$  that is constructed subject to a terminal condition such as  $p_T = p_T^*$  implies that the conditional variances are equal at  $T$ , but the estimation in this section does not explicitly account for this time dependence. However, the sampling distribution constructed by Monte Carlo simulation implicitly accounts for this since the same procedures are carried out there as for Standard and Poor's data.

<sup>27</sup> The sensitivity of results to the use of sample growth rates was checked in the simulations by repeating the analysis using the true (known) growth rate, and the results were essentially unchanged.

TABLE 3

RATIO OF CONDITIONAL STANDARD DEVIATION OF PRICE TO CONDITIONAL STANDARD DEVIATION OF  $p^*$  BY DIFFERENT CONDITIONING LAGS  $k$ , FOR STANDARD AND POOR'S SERIES 1926-79, AND SUMMARY STATISTICS OF THE DISTRIBUTION OF THIS RATIO IN 2,000 REPLICATIONS OF SIMULATED GEOMETRIC RANDOM WALK WITH SAMPLE SIZE 54

		SIMULATION							
LAG	STANDARD AND POOR'S RATIO	Number of Simulation Violations (Ratio > 1.0)	Ratio Mean*	Ratio Standard Deviation	Minimum Ratio	Maximum Ratio	50th (Median)	90th	95th
Rate $r = .05$ :									
$k = 1$	.46	0	.38	.15	.01	.92	.38	.57	.64
$k = 2$	.64	31	.53	.20	.01	1.23	.53	.79	.87
$k = 5$	.83	435	.78	.30	.02	2.09	.76	1.16	1.29
$k = 10$	1.17	816	.98	.41	.04	4.24	.91	1.48	1.76
Rate $r = .065$ :									
$k = 1$	.59	0	.41	.15	.02	.93	.41	.60	.66
$k = 2$	.81	32	.56	.20	.02	1.26	.57	.82	.90
$k = 5$	.96	495	.82	.29	.03	2.11	.80	1.19	1.30
$k = 10$	1.21	888	1.00	.38	.06	3.12	.95	1.49	1.73
Rate $r = .075$ :									
$k = 1$	.58	0	.43	.15	.02	.94	.43	.62	.68
$k = 2$	.79	34	.58	.20	.02	1.26	.59	.84	.90
$k = 5$	.92	528	.83	.28	.04	2.04	.83	1.19	1.30
$k = 10$	1.13	924	1.02	.37	.07	2.76	.97	1.48	1.69

NOTE.—The conditional variances at lag  $k$  are:  $\text{var}\{p_{t+k}/p_t|p_t\} = E\{[p_{t+k} - E(p_{t+k}|p_t)]^2\} = E\{[p_{t+k} - E(p_{t+k}|p_t)]^2\} = E(p_{t+k}^2|p_t) - E(p_{t+k}|p_t)^2$ , where  $E(p_{t+k}|p_t) = p_t(1 + g)^k$ . The growth rate  $g$  is estimated as  $g = \exp(\hat{\mu} + (\hat{\sigma}^2/2))$ , where  $\hat{\mu}$  and  $\hat{\sigma}$  are mean and standard deviation of the series  $\ln p_t - \ln p_{t-1}$ . The relevant processes are defined in table 1. No detrending is done here.

\* All summary statistics are for the distribution across 2,000 replications.

series do not violate the well-defined conditional variance inequalities implied by the valuation model (1). I now complete the argument by showing the empirical validity of the nonstationary price process used to derive these tests.

### C. Evidence on Nonstationarity of Prices, Earnings, and Dividends

This subsection applies the tests in Nelson and Plosser (1982) for nonstationarity of various macroeconomic variables to prices, earnings, and dividends used in variance bounds tests. First, autocorrelation functions for levels and first differences of random walks, and for residuals ("deviations from trend") from a regression of a random walk on time, are compared with the sample autocorrelations. Second, two different specifications of the autoregressive representation are tested directly for unit roots, as discussed in Fuller (1976) and Dickey and Fuller (1979).<sup>28</sup> The first specification is the simple autoregression (Dickey and Fuller 1979, p. 428, eq. 2.1)

$$Y_t = \mu + \rho Y_{t-1} + e_t, \quad (33)$$

where  $e_t$  is assumed i.i.d.  $N(0, \sigma^2)$  and  $\rho = 1.0$  under the null hypothesis. Fuller (1976, pp. 371, 373) tabulates empirical distributions for two test statistics for this model under the null hypotheses  $\rho = 1.0$  and  $\mu = 0.0$ . The first statistic is  $n(\hat{\rho}_\mu - 1)$ , where  $\hat{\rho}_\mu$  is the least-squares estimate of  $\rho$  in (33) and  $n$  is the sample size. The second test statistic,  $\hat{\tau}_\mu$ , is the " $t$ -statistic" under the null hypothesis  $\rho = 1$ .<sup>29</sup> The second specification (Dickey and Fuller 1979, p. 428, eq. 2.2) includes time as a regressor:

$$Y_t = \mu + \beta t + \rho Y_{t-1} + e_t. \quad (34)$$

The statistics are similar to those for (33) and are denoted  $n(\hat{\rho}_\tau - 1)$  and  $\hat{\tau}_\tau$  for the model (34) (Fuller 1976, pp. 371, 373).

#### 1. Stock Prices

The primary price data used here are Standard and Poor's annual composite stock price index for 1926–79 and quarterly composite

<sup>28</sup> Other procedures for testing for the existence of more than one unit root are discussed in Hasza and Fuller (1979) and applied in Meese and Singleton (1982). The hypothesis of multiple unit roots is rejected for the series examined here.

<sup>29</sup> For a given significance level the critical value of the statistic  $\hat{\tau}_\mu$  is larger (in absolute value) than for the usual  $t$ -distribution since the sampling distribution of  $\hat{\rho}_\mu$  is centered at values less than 1.0 in finite samples. Dickey and Fuller (1979, pp. 429–30) indicate that, if  $\mu \neq 0.0$  in (33), the statistic  $\hat{\tau}_\mu$  will imply acceptance of the hypothesis  $\rho = 1$  with probability greater than the nominal level, although they do not indicate the amount of discrepancy.

stock price index for 1947:I to 1978:IV (1980, pp. 134–37). Diagnostic plots of both levels and first differences show that the raw (nominal) data reflect price level changes in later periods. Consequently the series are deflated by the gross national product (GNP) implicit price deflator, and diagnostic checks indicate that this procedure seems adequate. Unless otherwise stated,  $p_t$  here refers to deflated prices.<sup>30</sup>

Tables 4 and 5 give results for autocorrelation and Dickey and Fuller (1979) tests, respectively. Section A in table 4 gives results for sample autocorrelations for the levels of seven series. The first three series are taken from Nelson and Plosser (1982, table 2). Series 1 and 2 are constructed as a random walk and a time-aggregated random walk,<sup>31</sup> and the autocorrelations are those expected in a sample of size  $T$  (here, 100). The third series is the log of nominal stock price. Series 4 and 5 (6 and 7) are deflated price and log of deflated price for Standard and Poor's annual (quarterly) series cited above. Section B in table 4 gives corresponding autocorrelations for first differences of the series, while section C gives autocorrelations for the residuals from a regression of the series on time, following Nelson and Kang (1981).

The major result from table 4 is that the autocorrelation functions for the stock price data show marked similarity to those for the constructed random walks. Several other results are also apparent. First, there seems little difference in the autocorrelation functions of the deflated price series,  $P_t/\text{GNP}_t$ , and its logarithm,  $\ln(P_t/\text{GNP}_t)$ . Second, the sample size affects the degree to which the first-order sample autocorrelation coefficient  $r_1$  is less than 1.0 in levels of the series. For the constructed random walk,  $r_1$  is 1.0 with infinite observations but 0.95 for sample size 100. For Standard and Poor's annual data (series 4, 5)  $r_1$  is approximately 0.90 ( $T = 54$ ), while for quarterly data (series 6, 7)  $r_1$  is 0.97 ( $T = 128$ ). Third, although first differences of Nelson and Plosser's nominal series (table 4, series 10) indicate large first-order autocorrelation ( $r_1 = 0.22$ , standard error .10), which is consistent with time aggregation, the deflated annual and quarterly Standard and Poor's data do not show such high first-order autocorrelation.<sup>32</sup>

<sup>30</sup> The stock price series in Nelson and Plosser (1982) is not deflated, and their results are reported in table 4 for comparison.

<sup>31</sup> That is, the series is constructed by averaging sets of observations from a random walk with a smaller observation interval than the resulting series. Working (1960) demonstrates that, as the number of shorter interval observations averaged to produce the resultant time-aggregated series becomes large, the first-order serial correlation in the latter series approaches .25.

<sup>32</sup> For the annual data,  $r_1$  is virtually zero, while the quarterly series shows  $r_1 = 0.14$  with a standard error of .09. The autocorrelation in the nominal series may reflect price level changes rather than temporal aggregation.

Section C in table 4 shows that not only do the stock price series match the constructed random walk data, but OLS regression of stock prices on time is very poorly specified. For series 19 ( $\ln P_t/\text{GNP}_t$ , annual data), the Durbin-Watson statistic is only 0.38,<sup>33</sup> reflecting the very high autocorrelation in the residuals. Nelson and Kang (1981) show that this is to be expected if a random walk is inappropriately regressed on time, and the results are consistent with those of Nelson and Plosser (1982).

Table 5 gives the results of the Fuller (1976) and Dickey and Fuller (1979) tests for prices. In no case is the null hypothesis  $\rho = 1$  rejected at the 10 percent level, and especially for the quarterly data the test statistics are well above the 10 percent critical value (rejection is indicated by small values of the statistics). Further, when time is included as a regressor (eq. [34] above), the null hypothesis  $\beta = 0$  is not rejected at conventional significance levels.<sup>34</sup> Although the intercept  $\hat{\mu}$  from (33) to (34) is not statistically far from zero in table 5, the implied economic magnitudes are very large, which is consistent with sample values of the slope coefficient  $\hat{\rho}$  less than 1.0 if the true coefficient equals 1.0. For example, the estimated intercept for series 2 (annual data,  $\ln P_t/\text{GNP}_t$ ) is 0.44, which implies an expected capital gain rate of over 0.44 per annum (in real terms). If one imposes the null hypothesis  $\rho = 1$  from the geometric random walk model (26),  $\hat{\mu}$  is given by the sample mean of the first differences in log of price ( $\nabla \ln p_t \equiv \ln p_t - \ln p_{t-1}$ ). For Standard and Poor's annual index, 1926–79,  $\hat{\mu}$  is 0.0095, and the point estimate of  $\sigma^2$  (the sample variance of  $\nabla \ln p_t$ ) is 0.048. This implies using (28) a (real) expected capital gain rate of 0.033 per annum, which is reasonable.

In short, tables 4 and 5 show that the random walk models (25) and (26) cannot be rejected for Standard and Poor's price series.

## 2. Earnings and Dividends

We examine whether the nonstationarity of dividends and earnings implied by (17) and (29) is supported empirically. The earnings per share and dividend per share series are Standard and Poor's annual series corresponding to the composite stock price index, 1926–79. Note that the accounting earnings series is only a proxy for the eco-

<sup>33</sup> Table 1 above gives the average Durbin-Watson statistic across 100 replications, for different sample sizes, of the regression of a (geometric) random walk on time. For samples of size 50 (table 4, series 19, has 54 observations), the average Durbin-Watson statistic is 0.375.

<sup>34</sup> For table 5, series 6 ( $\ln P_t/\text{GNP}_t$ , annual data), although the "*t*-statistic" is 2.26, this statistic does not have a true *t*-distribution for the sample size 54. For the corresponding quarterly series ( $T = 128$ ), the "*t*-statistic" is  $-1.05$ .

TABLE 4  
 SAMPLE AUTOCORRELATIONS FOR LEVELS, FIRST DIFFERENCES, AND DEVIATIONS FROM TIME TREND FOR RANDOM WALKS AND STOCK PRICES  
 (Annual and Quarterly Data)

SERIES*	PERIOD	T	AUTOCORRELATIONS						ADJUSTED $R^2$	DURBIN- WATSON
			$r_1^\dagger$	$r_2$	$r_3$	$r_4$	$r_5$	$r_6$		
A. Sample autocorrelations: levels:										
1. Random walk		100	.95	.90	.85	.81	.76	.70	...	...
2. Time-aggregated random walk		100	.96	.91	.86	.82	.77	.73	...	...
3. Log of price (ln $P_t$ ): annual	1871-1970	100	.96	.90	.85	.79	.75	.71	.10	...
4. Deflated price ( $P_t/P_t$ ): annual	1926-79	54	.91	.82	.81	.77	.71	.63	.13	...
5. Log of deflated price (ln $P_t/P_t$ ): annual	1926-79	54	.89	.79	.75	.71	.68	.64	.13	...
6. Deflated price ( $P_t/P_t$ ): quarterly	1947:I-1978:IV	128	.97	.93	.90	.86	.83	.79	.09	...
7. Log of deflated price (ln $P_t/P_t$ ): quarterly	1947:I-1978:IV	128	.97	.94	.91	.87	.84	.81	.09	...

B. Sample autocorrelations: first differences:

8. Random walk	Large	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
9. Time-aggregated random walk	Large	.25	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
10. $\ln P_t$ : annual	1871-1970	.22	-.13	-.08	-.18	-.23	.02	.10	.10	.10	.10	.10	.10	.10
11. $P_t$ /GNP: annual	1926-79	.54	-.31	.06	.15	.00	-.08	.13	.13	.13	.13	.13	.13	.13
12. $\ln(P_t/\text{GNP})$ : annual	1926-79	.54	.02	-.25	-.13	.01	.06	.13	.13	.13	.13	.13	.13	.13
13. $P_t$ /GNP: quarterly	1947:I-1978:IV	128	.14	-.09	.01	-.02	-.05	.09	.09	.09	.09	.09	.09	.09
14. $\ln(P_t/\text{GNP})$ : quarterly	1947:I-1978:IV	128	.13	-.07	.02	.02	-.01	.09	.09	.09	.09	.09	.09	.09
C. Autocorrelations of residuals ("deviations from trend")—Model:														
			$Y_t = \beta_0 + \beta_1 t + \epsilon_t$											
15. Random walk	61	.85	.71	.58	.47	.36	.27	.27	.27	.27	.27	.27	.27	.27
16. Random walk	101	.91	.82	.74	.66	.58	.51	.51	.51	.51	.51	.51	.51	.51
17. $\ln P_t$ : annual	100	.90	.76	.64	.53	.46	.43	.10	.10	.10	.10	.10	.10	.10
18. $P_t$ /GNP: annual	54	.81	.63	.55	.46	.33	.19	.13	.13	.13	.13	.13	.13	.13
19. $\ln(P_t/\text{GNP})$ : annual	54	.80	.57	.44	.33	.28	.22	.13	.13	.13	.13	.13	.13	.13
20. $P_t$ /GNP: quarterly	128	.95	.89	.83	.78	.72	.67	.09	.09	.09	.09	.09	.09	.09
21. $\ln(P_t/\text{GNP})$ : quarterly	128	.95	.90	.85	.80	.75	.70	.09	.09	.09	.09	.09	.09	.09

\* The source for series (1)-(3), (8)-(10), and (15)-(17) is Nelson and Plosser (1982) and references therein. The other series are Standard and Poor's annual (fourth quarter) and quarterly composite stock price indexes. GNP is the gross national product implicit price deflator.

†  $r_t$  is the  $n$ th-order sample autocorrelation coefficient.

‡ S.E. gives the approximate standard error of  $r$  for the sample size  $T$  under the null hypothesis of zero autocorrelation.



TABLE 5

TEST STATISTICS FOR SMALL SAMPLE TESTS FOR RANDOM WALKS IN STANDARD AND POOR'S ANNUAL PRICE SERIES (1926-79) AND QUARTERLY PRICE SERIES (1947:1-1978:IV)

SERIES	$\hat{\mu}$ ( <i>t</i> -Statistic)*	$\hat{\beta}$ ( <i>t</i> -Statistic)*	$\hat{\rho}$	$n(\hat{\rho} - 1)^\dagger$	<i>t</i> -STATISTIC‡ ( $H_0: \hat{\rho} = 1$ )	ADJUSTED $R^2$	Durbin- Watson	RESIDUALS			
								$r_1$ §	$r_2$	$r_3$	$r_4$
Model A: $Y_t = \mu + \rho Y_{t-1} + \epsilon_t$											
Annual (4th quarter):											
1. Deflated price ( $P_t/\text{GNP}_t$ ) <sup>  </sup>	6.63 (1.62)	...	.90	-5.04	-1.65	.83	1.96	.01	-.25	.10	.18
2. Log of deflated price (ln $P_t/\text{GNP}_t$ )	.44 (1.75)	...	.89	-5.57	-1.72	.80	1.84	.07	-.19	.03	-.08
Quarterly:											
3. $P_t/\text{GNP}_t$	2.54 (1.70)	...	.97	-3.77	-.16	.96	1.69	.15	-.08	.01	-.02
4. ln( $P_t/\text{GNP}_t$ )	.12 (1.92)	...	.97	-3.56	-.19	.97	1.72	.13	-.07	.02	.02
Model B: $Y_t = \mu + \beta t + \rho Y_{t-1} + \epsilon_t$											
Annual (4th quarter):											
5. $P_t/\text{GNP}_t$	3.42 (.81)	.28 (1.73)	.82	-9.22	-2.29	.84	1.93	.03	-.18	.20	.25
6. ln( $P_t/\text{GNP}_t$ )	.74 (2.60)	.006 (2.26)	.77	-11.59	-2.76	.83	1.86	.07	-.13	.18	.04
Quarterly											
7. $P_t/\text{GNP}_t$	2.75 (1.77)	-.02 (-.81)	.98	-2.45	-.82	.96	1.72	.14	-.10	-.00	-.04
8. ln( $P_t/\text{GNP}_t$ )	.09 (1.13)	-.0003 (-1.05)	.98	-2.03	-.76	.97	1.77	.11	-.10	-.00	-.01

\* Large sample *t*-statistics (in parentheses) under the null hypotheses that  $\mu$  and  $\beta$  equal zero.

† The 10 percent critical values for this statistic under the null hypothesis  $\hat{\rho} = 1$  are -10.7 (-11.0) for  $n = 50$  (100) under model A (i.e., with no time regressor) and -16.8 (-17.5) under model B (which includes time as a regressor) (Fuller 1976, p. 371).

‡ The 10 percent critical values under the null hypothesis  $\hat{\rho} = 1$  are -2.60 (-2.58) for  $n = 50$  (100) under model A and -3.18 (-3.15) under model B (Fuller 1976, p. 373).

§  $r_n$  is the  $n$ -th-order autocorrelation coefficient. The approximate standard errors are .13 and .09 for annual and quarterly data, respectively, under the null hypothesis of zero correlation. || GNP is the gross national product implicit price deflator.

nommic earnings series  $X_t$  in the Miller-Modigliani valuation models. The deflation procedure and nonstationarity tests used for prices are applied to the earnings and dividend series.

Table 6 gives the results from the Fuller (1976) and Dickey and Fuller (1979) tests, and the autocorrelation tests give similar results. Section A tests directly for unit roots in the simple autoregression (33), and the null hypothesis  $\hat{\rho} = 1$  is not rejected at even the 10 percent level for either earnings or dividends. Section B gives results for the autoregression (34), which includes time as an additional regressor. This model adds virtually no extra explanatory power over the simple autoregression (in terms of  $R^2$ ), and for the dividend series (series 7 and 8, table 6) the null hypothesis  $\hat{\rho} = 1$  is not rejected at the 10 percent level. However, the null hypothesis  $\hat{\rho} = 1$  is rejected at the 5 percent level for both earnings series (5 and 6) when time is included.

The earnings results produce an interesting question in interpretation and are similar to results for a dividend series that Shiller (1981c) relies on to conclude that dividends are stationary. When looking just at the simple earnings autoregression without time, the random walk model fits well. When time is included, although there is virtually no increase in  $R^2$ , the coefficient on time appears significantly different from zero and the coefficient on lagged earnings seems significantly less than one. On balance, the simple autoregression seems preferable. First, it is consistent with the results of other studies of earnings per share, including those based on individual securities.<sup>35</sup> Second, it is consistent with the price process established above and economically seems more reasonable than (34).

The evidence relied on in Shiller (1981c, 1983) for stationarity of dividends (and consequently prices) is more tenuous. He considers a combination of the Standard and Poor's data used in table 6 with earlier Cowles Commission data, which together extend from 1871 to 1978. For this series, he reports (1981c, p. 299, n. 7) that the autoregression of  $\log d_t$ , including time as a regressor, gives a coefficient on  $\log d_{t-1}$  of 0.807 and a standard error of .058, which has a probability value of .05 using Fuller's (1976) tabulations. On the basis of this result, he concludes (1983, p. 237) that "we can reject a random walk at the 5 percent level in favor of stationary fluctuations around a trend."

There are several problems with this interpretation. First, table 6

<sup>35</sup> See the references in n. 20 above. Note also the result in Watts and Leftwich (1977) that, although for particular samples some ARIMA models apparently fit better than the simple random walk (e.g., to accommodate the residual autocorrelation in table 6), there is little evidence that such models are better at prediction.

TABLE 6

TEST STATISTICS FOR SMALL SAMPLE TESTS FOR RANDOM WALKS IN STANDARD AND POOR'S ANNUAL EARNINGS AND DIVIDENDS SERIES (1926-79)

SERIES	RESIDUALS										
	$\hat{\mu}$ ( <i>t</i> -Statistic)*	$\hat{\beta}$ ( <i>t</i> -Statistic)*	$\hat{\rho}$	$n(\hat{\rho} - 1)^\dagger$	<i>t</i> -STATISTIC‡ ( $H_0: \hat{\rho} = 1$ )	ADJUSTED $R^2$	Durbin- Watson	$r_1$ §	$r_2$	$r_3$	$r_4$
	Model A: $Y_t = \mu + \rho Y_{t-1} + e_t$										
1. Deflated earnings ( $E_t/\text{GNP}_t$ ) <sup>  </sup>	.24 (.84)	...	.97	-1.54	-.51	.85	1.85	.06	-.14	-.14	-.11
2. Log of deflated earnings ( $\ln E_t/\text{GNP}_t$ )	.12 (1.30)	...	.93	-3.78	-1.16	.81	1.64	.17	-.09	-.24	-.09
3. Deflated dividends ( $D_t/\text{GNP}_t$ )	.27 (1.61)	...	.91	-4.85	-1.49	.81	1.60	.19	-.19	-.05	-.02
4. Log of deflated dividends ( $\ln D_t/\text{GNP}_t$ )	.12 (1.80)	...	.89	-6.09	-1.71	.77	1.63	.18	-.20	-.06	.01
	Model B: $Y_t = \mu + \beta t + \rho Y_{t-1} + e_t$										
5. $E_t/\text{GNP}_t$	.45 (1.72)	.05 (3.86)	.61	-19.86	-3.64	.88	1.72	.06	-.11	-.19	-.16
6. $\ln(E_t/\text{GNP}_t)$	.26 (2.81)	.01 (3.76)	.61	-20.04	-3.84	.86	1.58	.15	-.09	-.30	-.13
7. $D_t/\text{GNP}_t$	.35 (2.06)	.0009 (2.28)	.78	-11.25	-2.59	.82	1.59	.17	-.24	-.09	.01
8. $\ln(D_t/\text{GNP}_t)$	.13 (2.03)	.0004 (2.33)	.75	-12.71	-2.78	.79	1.62	.16	-.25	-.11	.02

\* Large sample *t*-statistics (in parentheses) under the null hypotheses that  $\mu$  and  $\beta$  equal zero.  
 † The 10 percent (5 percent) critical values for this statistic under the null hypothesis  $\hat{\rho} = 1$  are -10.7 (-13.3) for  $n = 50$  under model A (i.e., with no time regressor) and -16.8 (-19.8) under model B (which includes time as a regressor) (Fuller 1976, p. 371).  
 ‡ The 10 percent (5 percent) critical values under the null hypothesis  $\hat{\rho} = 1$  are -2.60 (-2.93) for  $n = 50$  under model A and -3.18 (-3.50) under model B (Fuller 1976, p. 373).  
 §  $r_i$  is the *i*-th-order autocorrelation coefficient. The approximate standard error is .13 under the null hypothesis of zero correlation.  
 || GNP<sub>*t*</sub> is the gross national product implicit price deflator.

shows that, even for the autoregression including time, the dividend series since 1926 does not reject the random walk. Although it is true that a longer data set gives greater power in such tests, it is likely that the very early data are less reliable than Standard and Poor's series. Second, the results for the longer dividend series are not as clear-cut as Shiller implies. Replication of his results (over 1871–1979) gives a value for  $n(\hat{\rho}_\tau - 1)$  of  $-21.61$ , which barely rejects at the .05 level, and a value for  $\hat{\tau}_\tau$  of  $-3.41$ , which in Fuller's tabulations is not significant at the .05 level. Further, the longer data do not reject the random walk in either prices or dividends using the simple autoregression (33) (without time as an additional regressor) at even the .10 level for either test statistic, and the price data do not reject the hypothesis  $\hat{\rho} = 1$  at the .10 level even when time is included.

### 3. Conclusion

In summary, the price data never reject nonstationarity, even for long time series, and although there are some cases in which nonstationarity in earnings or dividends appears rejected when time is included as a regressor, there is no rejection of nonstationarity of these series for the simple autoregression (33). Of course, even if the series were stationary, this does not indicate that the price series should be stationary because of the possible dividend (and accounting earnings) smoothing discussed above. In fact, time-series plots show that the dividend series since 1950 is much smoother than either the price or earnings series. This is consistent with the argument that earnings (and investments) are the fundamental variables and that a finite set of derived dividends may not be representative of the information used to set stock prices.<sup>36</sup> Even when smoothing is ignored, however, Sections IIIA and IIIB demonstrate that, once nonstationarity of prices is accounted for, valid variance bounds tests are not rejected in Standard and Poor's price and dividend data.

## IV. Conclusions

This paper demonstrates that reliance on plots of price and  $p_t^*$  to determine whether changes in expectations of future cash flows cause

<sup>36</sup> This argument casts some doubt on the procedures of Granger (1975), who combines a dividend smoothing model from Fama and Blahnik (1968) with a random walk in earnings, to generate predictions of future dividends for use with the dividend valuation model (1). What is not verified in his example is that the short-run properties of his smoothed dividend series are sufficient to derive the price process implied by his earnings model. If the smooth dividend stream is not representative of all future dividends, then the use in (1) of optimal forecasts based on the smooth process will not necessarily give the true rational price.

price changes is very misleading since by construction  $p_t^*$  will not correspond to  $p_t$  and will be much smoother than  $p_t$  if prices are set by (1) and the future is not known with certainty. Further, it is shown empirically that one cannot reject the hypothesis that prices are nonstationary and that the "gross violations" of the bound (4) that have been reported in the literature are consistent with incorrect application of estimation techniques that assume stationarity to nonstationary series. The conditional variance bounds (8) derived and tested here are valid if prices are nonstationary and are not violated for Standard and Poor's price and dividend series.

The implications of these results can best be seen with reference to the conclusions drawn in the literature from plots of price and  $p_t^*$  and the apparent violations of the inequality (4). Early conclusions were that stock prices cannot be reconciled with rational valuation models, as in Shiller (1981*b*, p. 422). Although Shiller (1981*b*, 1981*c*) recognizes that discount rates need not be constant, he argues that there is so little variation in the cash flow variables in such valuation models that discount rate movements must be very large if prices are rational. Moreover, he regards this possibility as at least counter to generally held views and states (1981*a*, p. 1) that "most people feel that stock price changes are due primarily to changing expectations about future dividends rather than changing rates of discount."

Attempts have been made to explain stock price movements in terms of nonconstant discount rates. The most influential work is that of Grossman and Shiller (1981),<sup>37</sup> whose primary claim, as noted by Shiller (1981*a*, p. 2), is that "most of the variability of stock prices might be attributed to information about consumption," which causes changes in discount rates. However, subsequent work has not been successful in extending their results. Hansen and Singleton (1983), for example, are able to explain only a small portion of the variability of stock prices in terms of nonconstant discount rates. Shiller (1981*a*) notes that, if price changes are driven by changes in expectations about aggregate consumption, then changes across assets should show a degree of contemporaneous correlation that is absent for the assets he examines. In general, even within the same industry and with very clean stock price data, there are wide cross-sectional differences in returns for any given period that seem difficult to reconcile purely in terms of changes in expectations of aggregate consumption.

Given the discouraging evidence on the ability of changes in expectations about consumption to explain changes in asset prices, Shiller

<sup>37</sup> See also LeRoy and LaCivita (1981), Shiller (1981*a*), Michener (1982), Hansen and Singleton (1983), Joerding (1983), Litzenberger and Ronn (1985), and Mehra and Prescott (1985).

(1981a, p. 40) suggests that it might be possible to develop a “psychological model of asset prices” that preserves large discount rate movements, although he argues that it seems equally plausible that there are “temporary fads or speculative bubbles.” He concludes: “If . . . the reader goes back to a rational expectations model in which information about potential dividend movements, rather than discount rate movements, causes stock prices to move, then since actual aggregate dividend movements of such magnitude have never been observed, what is the source of information about such potential movements? Can we be satisfied with a model which attributes stock price movements and their business cycle correlation to public rational expectations about movements in a variable which has, in effect, never yet been observed to move?”

This paper demonstrates a plausible solution to the apparent puzzle: The assertion that price changes cannot be attributed to changes in expectations of future cash flows, based on plots such as figure 3 and the results of tests of the bound (4), has simply not been established. Recall that figure 2 displays similar characteristics to Standard and Poor’s data in figure 3, yet by construction prices in figure 2 are set by the valuation model (1). Further, Kleidon (1983, chap. 6) shows that a large part of observed price changes can be associated with changes in expectations of future cash flows, using simple models and a few information variables.

Nevertheless, the question whether or not discount rates are constant as in (1) is a different issue. The variance bounds methodology may not be very powerful in detecting departures from constancy, as shown in Stock (1982). Further, even if the constant rate model performs relatively well empirically, there are still important theoretical questions about the conditions under which (1) will hold exactly. Although (1) does not require risk neutrality, the derivation of temporally constant expected rates of return for discounting expected cash flows—“risk-adjusted” discount rates—requires restrictive conditions in models of expected return that allow for risk aversion, such as the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) or more general models.<sup>38</sup>

One implication is that the construct  $p_t^*$  will not in general be the

<sup>38</sup> Although LeRoy (1973) demonstrates that discount rates are not necessarily constant with risk aversion, he does not show the converse. See Fama (1970a) and Constantinides (1980) for sufficient conditions for a constant discount rate (across time for a given security) with risk aversion, in the context of the CAPM. Note also that financial economists typically do not reserve the term “expected present value” model for (1) with constant discount rates but include the use of nonconstant risk-adjusted rates. More general models of asset pricing include Merton (1973), Rubinstein (1976), Lucas (1978), Breeden (1979), Brock (1982), and Grossman and Shiller (1982).

price that would prevail if investors had perfect foresight, and so the term "perfect-foresight" price is unfortunate. If investors were risk neutral, the rate  $r$  used to discount the uncertain flows in (1) would be the same as that used to discount the certain flows in (2), but in general the appropriate expected rates of return will be different. However, the analysis in this paper does not depend on whether  $p^*$  is truly the price that would prevail under perfect foresight or whether the definition (2) just gives the present value of the ex post dividends discounted at the (possibly risk-adjusted) rate  $r$  from (1).

The major impact of the variance bounds literature has been to suggest that virtually no stock price changes are related to changes in expectations of future cash flows and further that prices may be irrational. This impact has been widespread; for example, Arrow (1983) discusses the volatility of securities markets as compatible with "irrational judgements about uncertainty" (p. 13) and states (p. 12) that "[a] very rigorous analysis for the bond and stock markets (Shiller, 1979, 1981[b]) has shown the incompatibility of observed behavior with rational expectations models, at least in a simple form." At least one published paper explicitly presumes excess volatility in stock prices. Pakes (1985, p. 395, n. 3) states: "Note that the presence of the error term,  $\eta_{1,t}$ , implies that there may be more variance in stock market evaluations than can be justified by the variance in earnings (which accords with the results of LeRoy and Porter [1981] and Shiller [1981b])."

The results of this paper suggest that such modifications to our theories are, at best, premature.

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