

## WHY NEW ISSUES ARE UNDERPRICED\*

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This paper presents a model for the underpricing of initial public offerings. The argument depends upon the existence of a group of investors whose information is superior to that of the firm as well as that of all other investors. If the new shares are priced at their expected value, these privileged investors crowd out the others when good issues are offered and they withdraw from the market when bad issues are offered. The offering firm must price the shares at a discount in order to guarantee that the uninformed investors purchase the issue.

### 1. Introduction

Several years ago, Grossman (1976) showed that if one class of investors has superior information about the terminal value of an asset, the information can be read by anyone from the equilibrium price. This result produces a paradox. If anyone can infer private information from the equilibrium price, no one pays to collect information. Yet if no one collects information, the price reveals none, and an incentive emerges to acquire it.

The key to the paradox is the assumption of a noiseless environment. If noise is present in the equilibrium price, privileged information is secure. For the uninformed cannot be sure whether a high price reflects favorable information or extraneous factors, such as a change in risk aversion or a need for liquidity.

This paper takes an alternative approach. If price, which is observable, does not correspond to a unique level of demand, which is unobservable, then the main channel by which inside information is communicated to the market is destroyed. Until the channel is re-established, the informed investor has an opportunity to profit from his knowledge by bidding for 'mispriced' securities. In this way, the investor is compensated for his costly investigations into the asset's value, and obtains some remuneration for showing where capital should best be allocated.

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The setting for this model is the new issues market, in particular, the market for 'firm commitment offerings'. In a firm commitment offering, the firm and its investment bank agree on a price and quantity for the firm's first issuance of equity. Once the price is set, typically on the morning of the offer, no further adjustments are allowed. If there is excess demand, the underwriter rations the shares, sometimes exercising an 'overallotment option' which permits as many as 10% more to be sold. If there is excess supply, the offer concludes with unsold shares. The investment bank pays the firm for the surplus shares and disposes of them later at market prices. Each condition – excess supply or demand – is not observed until after the 'offering date'. Only then does the presence or absence of informed trading become apparent.

The new issue market resembles an auction, but the resemblance is not exact. Price is not determined by the bidding of investors. In particular, the investor with the highest valuation need not obtain the shares, even if the valuation exceeds the issuer's reservation (offer) price. That investor may simply not receive an allocation of rationed shares from the underwriter. Moreover, the issuing firm is both a bidder, who submits a price in consultation with the underwriting investment bank, and a seller, who exchanges an asset for cash. Nevertheless, the spirit of the model and its methodology belong to the auction literature.

This model is directed toward an explanation of an anomaly in the new issue market. New shares appear to be issued at a discount. Ibbotson (1975) tested this hypothesis and found, on average, an 11.4% discount in the offer price which disappeared within weeks in the aftermarket. Using a simpler model, Ibbotson and Jaffe (1975) found a 16.8% average excess return relative to the market. Both were unable to account for their findings. After suggesting several explanations, Ibbotson termed the phenomenon a 'mystery'.

The discount is a natural consequence of the present model, which incorporates asymmetric information and rationing. Ibbotson and Jaffe themselves notice that underpriced shares can be severely rationed. They mention that it is not uncommon for underwriters to receive, prior to the effective date, 'indications of interest' for five times the number of shares available. This phenomenon has an effect upon the uninformed investor. If an investor finds that he receives none of the underpriced issues due to rationing brought on by informed demand, and all of the overpriced issues, then the investor revises downwards his valuation of new shares. He does not participate in the new issue market until the price falls enough to compensate for the 'bias' in allocation.

The analysis shows that the equilibrium offer price includes a finite discount to attract uninformed investors. This result is not a foregone conclusion. It is not immediately clear what advantage accrues to the issuer from uninformed participation. Nor is it clear if any discount is sufficient to attract them to the offering. It is conceivable that reducing the offer price could elicit greater

informed demand, exacerbate the bias, and further disadvantage the uninformed.

The analysis also shows that the optimal offer price is but one of a continuum of feasible prices. Different prices have different levels of uninformed investment and different probabilities of receiving rationed shares. Contrary to intuition, a small change in price does not produce a large uninformed response, even as the number of investors goes to infinity. The limiting demand schedule is easy to compute and does not depend upon the degree of risk aversion of the investors.

## 2. Relation to other work

Ritter (1984) has developed an implication of the current model and applied it to the 'hot issue' market of 1980. In general, the greater the uncertainty about the true price of the new shares, the greater the advantage of the informed investors and the deeper the discount the firm must offer to entice uninformed investors into the market. Ritter tested to see whether the predictable occurrence of market cycles in which initial offerings are deeply discounted could be explained as a change in the composition of the firms going public. The hypothesis is that during one phase, the initial uncertainty about firm values is low while during the other the uncertainty is high. While Ritter finds a significant statistical relation between the price variability of an issue in the aftermarket (which serves as a proxy for initial uncertainty) and the size of the discount, he concludes that the hot issue market of 1980 is attributable to another factor, the sudden appearance of natural resource firms going public.

In addition to Ibbotson and Ritter, several other authors find new issues to be underpriced, notably, Reilly (1977), Logue (1973), McDonald and Fisher (1972) and Reilly and Hatfield (1969). Among those offering explanations for the underpricing phenomenon, Baron (1980) argues that the discount is due to the superior information of the investment banker who sets the price and distributes the issue. Later, Parson and Raviv (1985) argue that the discount is a result of asymmetric information among investors, and they explain how both seasoned and unseasoned offerings are, on average, underpriced.

## 3. The model

Consider a market in which there are two assets available for investment. One is a safe asset whose return is normalized to 1. The other is an asset whose value per share,  $\tilde{v}$ , is uncertain. It is the latter asset which is being issued. The issuer pre-selects an offer price,  $p$ , and an offer quantity,  $Z$  shares. Once selected, the issuer receives offers to buy in quantities that vary according to the investor. Because no re-adjustment of price or quantity is allowed, the issuer can experience demand in excess of supply. In this case, the issuer can

fill only a fraction of the incoming orders. Thus, in the new issue market, the probability that an order is filled can be less than one.

When oversubscription occurs, it is assumed to result exclusively from large orders placed by investors who have favorable information about the prospects of the offering. This privileged segment of the market is called 'the informed'. All other investors, including the issuer, are called 'the uninformed'.

There are several reasons for regarding the issuer as uninformed, notwithstanding the fact that the firm and its agent, the investment banker, know a considerable amount about the company's future. First, the firm gives up its informational advantage by revealing its proprietary knowledge to the market. The firm discloses 'material information' about its plans and activities directly through the prospectus. Indirectly, the firm and the underwriter disclose their assessment of the firm's financial future through how aggressively they price the issue relative to 'comparable' offerings. Indeed, one role of the investment banker is to certify, by means of his reputation, that the proposed price accurately reflects the firm's prospects [see Beatty and Ritter (1986)]. Second, even though the firm and its agent know more than any single individual in the market, they know less than all the individuals in the market combined. While the investment banker is the one agent best suited to price the offering, his information and expertise are inferior to the pooled talents and knowledge of all the agents. Some individuals may have inside information about a competitor that could have a significant impact upon the firm's product. Others may know better than the firm or the investment banker the appropriate rate to discount the firm's cash flows in the capital market. Indeed, it is almost tautological that the firm and its banker are at a considerable informational disadvantage relative to the market as a whole. It is not unusual for the price set by the underwriter to be off by more than 20% when compared to the price established at the end of the first trading month. In fact, if the initial returns from the offer price to the closing price on the first trading day are averaged across all the firms going public in a given month, there are 18 months between 1/75 and 1/81 in which the average exceeds 20%. There are 5 months in which the average exceeds 40%. The firm and its underwriter, then, seem to be in substantial disagreement with the market over what the stock is 'truly' worth.

To emphasize the informational advantage which the market enjoys over the firm and the underwriter, it is assumed that:

- A.1. The informed investors have perfect information about the realized value of the new issue.

In addition:

- A.2. Informed investors cannot borrow securities or short-sell. They cannot sell their private information.

The first part of the second assumption is true almost by definition. To sell the shares short, an investor must physically borrow them, which is impossible on or before the issue date unless the shares are received from the firm itself. If the issuer, however, loans the stock, it is guilty of pre-issuing the offer and circumventing the securities laws.

The other assumptions are:

- A.3. Informed demand,  $I$ , is no greater than the mean value of the shares offered,  $\bar{v}Z$ .
- A.4. Uninformed investors have homogeneous expectations about the distribution of  $\bar{v}$ .
- A.5. All investors have the same wealth (equal to 1) and the same utility.

In addition to these five assumptions, the investment bank is implicitly regarded as an invisible intermediary. The firm is assumed to dictate the price of the offering, not the underwriter. In addition, the firm rather than the investment bank bears the risk of having the issue undersubscribed.

By A.1, the informed submit orders for the new shares whenever the realized value per share,  $\bar{v}$ , exceeds the offer price,  $P$ . By A.2, the informed order to the full extent of their wealth (equal to 1). And by A.3, when the informed order, they order a constant dollar amount:

$$\begin{aligned} I & \text{ if } p < \bar{v}, \\ 0 & \text{ if } p > \bar{v}. \end{aligned}$$

Unlike the informed, the uninformed, who are  $N$  in number, cannot predicate the size of their order upon the realization of  $\bar{v}$ . By A.4 and A.5, each uninformed investor wants to submit the same fraction,  $T$ , of his wealth (equal to 1) for the new issue. Since short-selling is impossible, each investor submits the positive share  $T^* = \max(0, T)$ . The combined dollar demand of the informed and uninformed is

$$\begin{aligned} NT^* + I & \text{ if } p < \bar{v}, \\ NT^* & \text{ if } p > \bar{v}. \end{aligned}$$

Since the demand fluctuates according to whether  $\bar{v}$  is above or below  $p$ , the issuer must experience either excess supply or excess demand in one of the two states. In the state  $\bar{v} > p$ , let the probability that an order is filled be denoted  $b$ . If  $\bar{v} < p$ , designate the probability  $b'$ . To relate  $b$  and  $b'$  to fundamental magnitudes, a particular mechanism for allocating rationed shares must be devised.

The incoming orders are assigned a lottery number upon arrival. These numbers are drawn at random, and the corresponding orders are filled in their entirety. The drawings conclude when there are either no more orders or no more shares. Clearly, under this rationing scheme, the probability that an order is filled is independent of its size, as implicitly assumed in the definition of  $b$  and  $b'$ .

In some countries, for example, England, the underwriter *must* allocate the shares in an even-handed fashion. In the U.S., however, the underwriter has more discretion. This latitude leads to a common complaint that domestic underwriters tend to favor their established customers. To the extent that these customers are better informed than the rest, this arrangement exacerbates the bias against the uninformed and leads to larger discounts.

The discretionary power of the underwriter, however, holds some benefit for the uninformed investor. If underwriters deny allocations to customers who quickly traded out of their positions at a large gain in the past, they diminish the bias against the uninformed and decrease the size of the discount. Indeed, one might speculate that the successful underwriter is the one who can best discriminate among potential investors, giving first priority to the uninformed and second place to informed customers of longstanding who can rebate some of their profits via commissions on future trades.

If rationing occurs, the value of the issue equals the value of the orders filled, plus some small excess if the last order chosen cannot be totally accommodated. Upon ignoring the small 'round-off' error, we have

$$\tilde{N}_u T^* + \tilde{N}_i = pZ \quad \text{if } b < 1,$$

where  $\tilde{N}_u$  is the number of uninformed orders filled and  $\tilde{N}_i$  is the number of informed orders filled.

Taking expectations,

$$bNT^* + bI = pZ \quad \text{if } b < 1,$$

or

$$b = \min\left(\frac{pZ}{NT^* + I}, 1\right). \quad (1)$$

Similarly,

$$b' = \min\left(\frac{pZ}{NT^*}, 1\right) \quad (2)$$

Observe that  $b < b'$ , which says directly that the probability of receiving an allocation of an underpriced issue ( $\tilde{v} > p$ ) is less than or equal to the probability of receiving an allocation of an overpriced issue ( $\tilde{v} < p$ ). This bias in

Table 1

Terminal wealth of investor as a function of the aftermarket value of the new issue and the probability of obtaining an allocation.

	Aftermarket value <sup>a</sup>			
	$\bar{v} > p$ (underpriced)		$\bar{v} < p$ (overpriced)	
	yes	no	yes	no
Wealth	$p^{-1}\bar{v}T + (1 - T)$	1	$p^{-1}\bar{v}T + (1 - T)$	1
Probability	$b_e p(\bar{v} > p)$	$(1 - b_e)p(\bar{v} > p)$	$b'_e p(\bar{v} < p)$	$(1 - b'_e)p(\bar{v} < p)$

<sup>a</sup>Aftermarket value is the price,  $v$ , realized on the first trade; the aftermarket price differs from the offering price,  $p$ , according to whether the issue is underpriced ( $v > p$ ) or overpriced ( $v < p$ ). The probabilities of these two events from the viewpoint of the uninformed investor are denoted  $p(v > p)$  and  $p(v < p)$ , respectively. Given the issue is underpriced, the probability of an allocation is  $b_e$ ; given the issue is overpriced, the probability of an allocation is  $b'_e$ . The uninformed investor has unit wealth initially, and chooses a fraction,  $T$ , to invest in the new issue.

allocation causes the uninformed investors to revise downward their valuation of the new shares. Therefore, to attract uninformed investors to the offering, the issuer must price the shares at a discount, which can be interpreted as compensation for receiving a disproportionate number of overpriced stocks.<sup>1</sup>

When uninformed investors decide on the fraction of their wealth to be placed in the new issue, they base the decision upon their prior beliefs regarding  $b$  and  $b'$ . To emphasize that prior expectations are involved,  $b$  and  $b'$  are temporarily subscripted by 'e'. Uninformed investors calculate  $T$  by maximizing their expected utility of terminal wealth. Table 1 presents the investor's terminal wealth as a function of the aftermarket value of the new issue and the probability of receiving an allocation. In table 1, if an investor submits an order which is not transacted because of rationing, the order is transformed into an equal dollar amount of the safe asset.

From the table, the uninformed investor has the expected terminal utility:

$$\begin{aligned}
 & b_e p(\bar{v} > p) E[U(1 + T(p^{-1}\bar{v} - 1)) | \bar{v} > p] \\
 & + b'_e p(\bar{v} \leq p) E[U(1 + T(p^{-1}\bar{v} - 1)) | \bar{v} \leq p] \\
 & + [1 - b_e p(\bar{v} > p) - b'_e p(\bar{v} \leq p)] U(1).
 \end{aligned}
 \tag{3}$$

<sup>1</sup>For an investor to experience a biased allocation, it is not necessary that others be perfectly informed. It is sufficient that aggregate demand be more informative than his personal observation. For example, let investor  $i$  derive a noisy estimate,  $\bar{v}_i$ , of the true value per share,  $v$ . The estimate is  $\bar{v}_i = v + \tilde{\epsilon}_i$ . Suppose  $i$ 's demand is an increasing function of the ratio  $(\bar{v}_i/p)$ . If the errors,  $\tilde{\epsilon}_i$ , are independent and the market is large, aggregate demand is a non-stochastic, increasing function of  $(v/p)$ . Thus, underpriced shares are more strictly allocated than overpriced shares.

Therefore, the optimal  $T$  satisfies the first-order condition

$$(b_e/b'_e)p(\bar{v} > p)E[U'(1 + T(p^{-1}\bar{v} - 1))(p^{-1}\bar{v} - 1)|\bar{v} > p] \\ + p(\bar{v} \leq p)E[U'(1 + T(p^{-1}\bar{v} - 1))(p^{-1}\bar{v} - 1)|\bar{v} \leq p] = 0.$$

A small insight into the economics of the offering process can be extracted from the form of the first-order condition. As far as the investor is concerned, it is not rationing *per se* which lowers his estimate of the value of the offering when he obtains an allocation. If rationing occurs to the same degree for both underpriced and overpriced issues, uninformed demand is the same as if there is no rationing. Rather, it is the *bias* in rationing good issues relative to bad issues which is important, the bias being measured by the ratio  $(b_e/b'_e)$  in the optimality condition.

To finish the description of the equilibrium, it only remains to require that the expectations of the investors be rational. Investors' beliefs about the chances of being dealt a good or bad offer must equal the actual probabilities which arise from the allocation mechanism. Upon equating investors' beliefs to the actual outcomes given by eqs. (1) and (2), the complete equilibrium is

$$b = \min\left(\frac{pZ}{NT^*(b/b', p) + I}, 1\right), \quad (4)$$

$$b' = \min\left(\frac{pZ}{NT^*(b/b', p)}, 1\right), \quad (5)$$

$$0 = (b/b')p(\bar{v} > p)E[U'(1 + T(p^{-1}\bar{v} - 1))(p^{-1}\bar{v} - 1)|\bar{v} > p] \\ + p(\bar{v} \leq p)E[U'(1 + T(p^{-1}\bar{v} - 1))(p^{-1}\bar{v} - 1)|\bar{v} \leq p], \quad (6)$$

$$T^*(b/b', p) = \max(0, T(b/b', p)).$$

In examining the equilibrium close attention is paid to how the uninformed change their demand in dollar terms as the offer price changes. The major question is whether uninformed investment increases as the offer price is reduced. This question involves more than whether the price is in the elastic



portion of the demand curve. The additional consideration is that uninformed investment,  $T(b, p)$ , depends not only upon the price but also upon the probability of receiving an allocation of underpriced shares. The probability of receiving an allocation *declines* as the price is lowered and, hence, counteracts the usual effect of price on demand.

The reason why the probability declines is that, as the issue is made less expensive, the informed investors can purchase a larger fraction of it. Other things equal, the informed become relatively more influential and, as a consequence, worsen the bias against the uninformed. To see this point analytically, hold uninformed investment,  $T(b, p)$ , fixed while decreasing the price. Then the denominator of eq. (4), which determines  $b$ , does not change while the numerator declines. Thus, the probability,  $b$ , must also decline. As a result, while uninformed demand may be stimulated by a decrease in the offer price, it is diminished by the smaller probability of obtaining desirable shares.

Establishing that uninformed investment increases with a price reduction is essential. There are two principal reasons why a company enters the new issue market. One reason is to refinance the firm. After several years of successful operation, the founders, venture capitalists and employees holding stock options have a considerable amount of wealth invested in the enterprise. Not only are they interested in adding some liquidity to their investments, they are also anxious to diversify their portfolios. The same motive applies to older companies with employee stock ownership plans. As employees retire, they want to diversify their pension assets and convert their holdings into cash in order to consume. Since selling shares back to the company requires the firm to use up valuable funds and negotiate with employees about the terms of repurchase, a simpler procedure for all the parties involved is to take the company public.

A second reason to go public is to obtain new funds. Having gone several rounds with the banks and the venture capitalists, a firm may have no alternative but to seek funds in the public market to finance new investment. Even if bank financing or venture funding is available, the equity market allows larger sums to be raised more efficiently, without the need for complex covenants and restrictions.

The first motive for going public is risk aversion on behalf of the owners, pensioners and financial backers of the firm. The second motive is the desire to take advantage of a positive net present value investment opportunity. In both cases, the firm faces a tradeoff. If uninformed demand increases as the price is reduced, the lower the price, the larger the payment guaranteed to the firm from the offering. This guarantee offers protection to risk-avoiding claimants who otherwise are exposed to declines in the value of their assets. Moreover, it assures the firm which is contemplating an investment opportunity that the funds which are necessary to undertake the project will be available. The task facing the issuer, therefore, is to trade the guaranteed payment against the

expected proceeds from the offering. That is, the issuer must trade higher minimum proceeds for a lower average take.

#### 4. The opportunity set facing the issuer

Before investigating whether uninformed demand slopes downward like a proper demand curve, the first proposition to establish is that an equilibrium exists. The chief concern is whether there are any sets of beliefs that are consistent with the actual probabilities of receiving an allocation.

A useful heuristic is to consider what happens when the number of investors is very large. In this case, the risky asset represents a small fraction of each investor's total wealth. Since individuals are approximately risk-neutral with respect to small gambles, any uninformed investor who buys the initial public offering expects a return which is close to the risk-free rate.

The fact that an uninformed investor earns approximately the risk-free rate in a large market essentially determines his chances of receiving an allocation of good shares. If an uninformed investor submits a bid, his expected profit is<sup>2</sup>

$$bp(\tilde{v} > p)E(\tilde{v} - p|\tilde{v} > p) + p(\tilde{v} < p)E(\tilde{v} - p|\tilde{v} < p).$$

Upon requiring zero abnormal profits, we have

$$b \equiv b_0(p) = \frac{p(\tilde{v} > p)E(\tilde{v} - p|\tilde{v} > p)}{p(\tilde{v} < p)E(p - \tilde{v}|\tilde{v} < p)}.$$

This is the smallest probability an uninformed investor will tolerate of obtaining rationed shares before withdrawing from the new issue market, given the offering price is  $p$ . The function,  $b_0(p)$ , therefore, is called the 'zero demand probability'. Since, for large markets, each uninformed investor is on the verge of demanding zero, the resulting probability of receiving an allocation should be close to the zero demand probability. For large markets, at least, the existence of a consistent set of beliefs is guaranteed because  $b_0(p)$  depends only upon the offer price and not upon the particulars of the investor's utility function.

For markets of arbitrary size, the existence of a consistent set of beliefs – called  $b(p, N)$  to emphasize the dependence upon both the price and the number of investors – is proven in the following theorem:

*Theorem 1.* Let  $0 < p < \bar{v}$ . Then

$$b = \min\left(\frac{pZ}{NT^*(b, p) + I}, 1\right)$$

has the unique solution  $b(p, N)$ . It satisfies  $b(p, N) > b_0(p) > 0$ .

<sup>2</sup>See table 1. We assume that bad shares are not rationed (i.e.,  $b' = 1$ ).

*Proof.* See appendix.

The following lemma confirms the conjecture that as the number of investors tends to infinity, their beliefs about the chances of being rationed converge to the ‘zero demand probability’. The lemma also shows that when the number of investors is large, the zero demand probability gives an accurate picture of how beliefs *change* with the offer price.

*Lemma.* Let  $(\partial^2/\partial b \partial p)T(b, p)$  and  $(\partial^2/\partial b^2)T(b, p)$  be continuous in the region  $0 < b < 1$ ,  $0 < p < \bar{v}$ . Then, the following limits hold uniformly:

$$\lim_{N \rightarrow \infty} b(p, N) = b_0(p),$$

$$\lim_{N \rightarrow \infty} \frac{d}{dp} b(p, N) = \frac{d}{dp} b_0(p).$$

*Proof.* See appendix.

Our interest in the function  $b(p, N)$  stems from a desire to understand how uninformed demand changes in response to a change in the offer price. If the probability of obtaining good issues does not fall too much as the offer price declines, then uninformed demand increases. The fact that the probability  $b(p, N)$  converges uniformly, with its derivatives, to  $b_0(p)$  simplifies the study of uninformed demand for large markets because the zero demand probability can be so easily computed. This enables us to prove:

*Theorem 2.* For large markets and any price below  $\bar{v}$ ,

$$\frac{d}{dp} T(b(p, N), p) < 0.$$

*Proof.* See appendix.

We can now completely describe the opportunity set. Suppose the market price is initially set equal to the mean value of the shares,  $\bar{v}$ , and the informed are not numerous enough to buy the entire issue, even if they wanted to. At this price, there is no rationing. The informed orders do not cause rationing by themselves, and the uninformed are unexcited by the chance to earn the risk-free rate on a small but risky investment. As the price is lowered, the uninformed become more interested, and they start to submit orders. At some critical price, the issue is fully subscribed in the state of the world where the informed know the issue is worth purchasing (‘the good state’). At this price, uninformed demand plus informed demand exactly equals the dollar value of the offering.

Further reductions in price elicit even larger uninformed orders, according to Theorem 2. The uninformed and informed are now competing for shares whose offer price is growing smaller as the value of their orders is growing larger. The result is that all orders must be rationed more strictly in the good state of the world.

As a result of the uninformed, demand is also growing in the state of the world where the informed do not find the shares worth purchasing ('the bad state'). At some point the price is so low that the uninformed by themselves can fully account for the issue. This is called the 'full subscription price' – the price at which the issuer can rely on selling all the shares in the bad state, as well as in the good state.

For prices lower than the full subscription price, continued growth in uninformed demand causes rationing in both states but the amount of rationing in the good state *relative* to the bad state declines. Indeed, as the uninformed begin to dominate the market, the chances of being rationed in each state become the same. As a result, the informativeness of receiving an allocation tends to zero, since the market realizes that a successful bid does not necessarily mean a lack of interest on the part of the informed.

This effect produces a curious result. The more nearly equal the chances are of receiving an allocation in the good and bad state, the larger is the demand of the uninformed, who care only about the *bias* in the rationing. The larger the uninformed demand, however, the smaller the bias, which calls forth even greater uninformed demand, *ad infinitum*. Uninformed demand literally *explodes* when the price goes below the full subscription amount, as the example below demonstrates.

Before considering an example, it is important to verify that the 'full subscription' price always exists and to have some simple formula for computing it when the number of investors is large. By definition of the full subscription price,  $p_f$ ,

$$p_f Z = NT(b(p_f, N), p_f).$$

Upon substituting this into the defining relation for  $b(p, N)$ , eq. (4), we have

$$b(p_f, N) = p_f Z / (p_f Z + I).$$

For large markets, the probability of receiving an allocation,  $b(p, N)$ , is uniformly close to the 'zero demand probability',  $b_0(p)$ . The full subscription price, then, must be close to the solution of

$$b_0(p) = pZ / (pZ + I), \tag{7}$$

which can always be shown to exist.

### 5. An example

This section verifies some of the important assertions made in the preceding section. The first assertion is that uninformed demand increases as the offer price is lowered. Second, the probability of receiving an allocation of underpriced shares converges to the 'zero demand probability' as the market grows larger. Third, the full subscription price is easy to calculate and nears the specified limit. Finally, demand 'explodes' when the offer price goes below the full subscription point.

For this example,

- the value of the issue per share,  $\bar{v}$ , is uniformly distributed on the interval  $(0, 2\bar{v})$ ;
- investors have identical quadratic utilities,

$$U(w) = w - gw^2/2.$$

The first step is to calculate the uninformed demand from eq. (6). Since the utility is quadratic, the equation is linear in  $T$  and easily yields the solution

$$T(b/b', p) = 3 \frac{(1-g)}{g} \left\{ \frac{(b/b')(2\bar{v}/p - 1)^2 - 1}{(b/b')(2\bar{v}/p - 1)^3 + 1} \right\}. \tag{8}$$

Eqs. (4) and (5) are not in a form that can be readily solved. They can be replaced by the equivalent relations

$$b = \min \left[ \frac{pZ}{NT^*(b, p) + I}, 1 \right] \quad \text{if } b' = 1, \tag{9}$$

$$(b/b') = \frac{NT^*(b/b', p)}{NT^*(b/b', p) + I} \quad \text{if } b' < 1. \tag{10}$$

Observe from the form of  $T$  in eq. (8) that eqs. (9) and (10) lead to quadratics in  $b$  and  $(b/b')$ , for a given  $p$ . The eqs. (9) and (10) are, accordingly, straightforward to solve.

Table 2 calculates the uninformed demand and the probability of receiving an allocation under two sets of assumptions about the parameters. In each case, the uninformed have the expectation that overpriced shares are not rationed, an expectation which is analytically equivalent to  $b' = 1$ . Later, these expectations will be examined to see whether they can be maintained over the whole range of prices and, if so, whether other expectations also make sense.

The table confirms that as the offer price falls, uninformed demand increases both in absolute dollar amount and as a percentage of the market value of the issue. The increase occurs notwithstanding the fact that good issues are harder to get, i.e.,  $b$  is decreasing.

Table 2

Uninformed demand and probability of an allocation of underpriced shares to an uninformed investor, as a function of offer price and number of investors; large (A) and small (B) markets.

Offer price as percentage of mean price/share $100 \times p/\bar{v}$	Uninformed demand as percentage of mean value of issue $100 \times NT^*/\bar{v}Z$		Uninformed demand as percentage of market value of issue $100 \times NT^*/\bar{p}Z$		Probability of an allocation of underpriced shares $b$		Zero demand probability $b_0(p)$
	(A)	(B)	(A)	(B)	(A)	(B)	
	95%	14%	17%	15%	18%	84%	86%
90	27	25	30	28	71	72	67
85	41	30	50	35	59	66	55
80	56	34	70	43	51	60	44
75	71	40	96	53	43	53	36
70	86	47	123	67	38	47	29
65	100	55	154	85	33	42	23
60	114	62	190	103	29	37	18
55	129	71	235	129	25	32	14
50	143	80	286	160	21	28	11

If the discount is kept fixed while the size of the market increases, the probability of receiving an allocation of underpriced shares falls. For instance, when the offer price is 80% of the mean price per share, the chances of being rationed go down from 60% to 51% as the market goes from small to large. This result is found along the row corresponding to the 80% offer price in the leftmost column of table 2. By moving from small to large under the heading  $b$ , the probabilities decrease from 60% to 51%. Eventually, as the market becomes infinitely large, the chances approach the 'zero demand probability', shown in the rightmost column.

As the name implies, the 'zero demand probability' makes the demand in eq. (8) zero:

$$b_0(p) = \frac{1}{(2\bar{v}/p - 1)^2}.$$

The usefulness of this schedule extends beyond the calculation of the asymptotic probabilities of obtaining a share of a good issue. According to eq. (7), solving

$$b_0(p) = \frac{1}{(2\bar{v}/p - 1)^2} = \frac{p}{p + \bar{v}}$$

yields the price at which uninformed demand is sufficient to subscribe the

entire offering. That solution involves a 20% discount from the mean. Although the solution is exact only for markets with infinitely many uninformed investors, it provides an approximation to the full subscription price for markets of any size. For instance, in the large market case, the full subscription price is 74% of the mean price per share. At this price, uninformed demand is 100% of the market value of the issue. The price, accordingly, is obtained from table 2 by interpolating between the uninformed demands in the center column until one is found which equals the market value of the issue. For large markets, equality occurs somewhere between 75% and 70% of the unconditional mean price per share; say, at 74%. Thus, a 26% discount, rather than a 20% one, is needed to insure that all the shares are sold in every state of the world.

If, for each offer price, we graph the probability of receiving an allocation of underpriced shares when the market is 'large', we obtain curve A, B shown in fig. 1. Observe that the solid segment stops at point B. Below point B, the offer price is so low that uninformed demand exceeds the market value of the offering (see the large market column in the center of table 2). In this region, the shares are always oversubscribed. This contradicts our explicit assumption in forming table 2, that investors do not expect overpriced shares to be rationed. As a result, points along the dotted segment can never be observed. The question remains, then, what happens if the issuer insists on lowering the price below that corresponding to point B? If the issuer insists on having such large discounts, investors must revise their expectations and submit different orders.

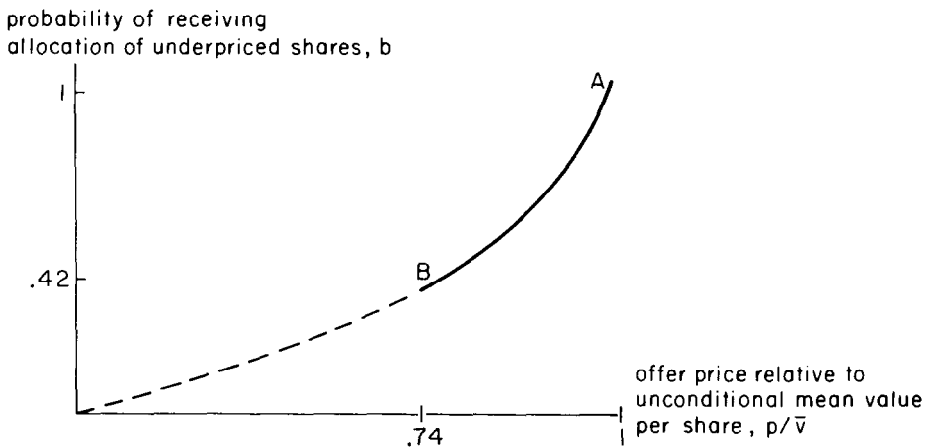


Fig. 1. Probability of receiving an allocation of underpriced shares as function of the offer price per share. For prices less than 74% of the mean value per share, shown as the dotted portion of the graph below point B, the probability of receiving an allocation of overpriced shares is less than one.

Earlier, we observed that uninformed demand depends only upon the bias in allocating shares. We measured this bias by the probability of being rationed in the good state relative to the probability of being rationed in the bad state ( $= b/b'$ ). Upon graphing the 'relative' probability of being rationed versus the offer price, we obtain a complete picture of the new issue market. (See fig. 2.)

Fig. 2 looks much like fig. 1. The segment A,B is the same since only underpriced shares are rationed along its length. As the price drops below B, however, oversubscription occurs for bad shares as well as good shares. Demand by the uninformed actually increases because good shares become *relatively* easier to obtain than bad shares. As uninformed demand rises, the bias in allocating good shares relative to bad shares is further attenuated, raising uninformed demand still more. Uninformed demand jumps discontinuously, which is reflected in the discontinuous change from B to C when the price falls incrementally below the point of full subscription.

Analytically, the multiplicity of expectations at a given price arises from the fact the expectations are defined by two quadratic equations. Eq. (9), which defines segment A, B, always has one positive root, while eq. (10) generally has two.

It is interesting to speculate whether there is any connection between the multiplicity of solutions and observed behavior in the new issue market. A very tentative connection can be made with the hot/cold cycles of initial public

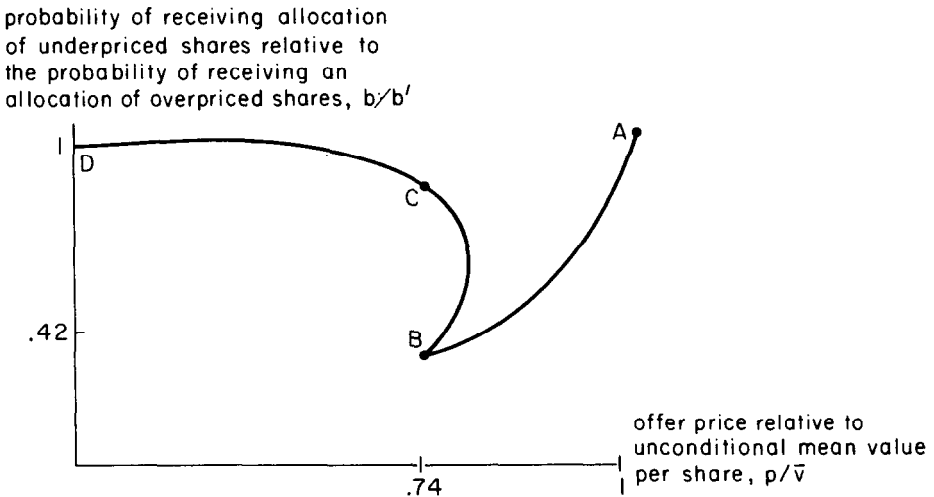


Fig. 2. Relative probability of receiving an allocation of underpriced shares as function of offer price per share. Points B and C correspond to two distinct equilibria at the same offer price. At point C, uninformed demand is significantly larger than at point B, causing both underpriced and overpriced shares to be rationed.



offerings. During the cold issue cycles, discounts are large, but the number of offerings are few. Several offerings are even undersubscribed. This is exactly what happens along the lower branch from A to B (see fig. 2). During the hot issue cycles, however, demand is heavy and discounts are smaller than in the cold cycle. This behavior corresponds to the upper branch from B, C, D where all offerings are fully subscribed.

## 6. The optimal offer price

As mentioned in section 3, there are two motives for going public. Here, we will investigate only one: risk aversion on the part of the founders, employees and financial backers of the firm.<sup>3</sup> Given this motive, the offer price cannot exceed the unconditional mean value of the shares,  $\bar{v}$ . If the price is greater than  $\bar{v}$ , the uninformed never submit an order. The issuer sells exclusively to the informed. Thus, when the shares are bad, the issuer ends up holding them, and when the shares are good, the issuer sells them off. No matter what the degree of risk aversion, the 'owners' are clearly better off to retain the shares rather than sell in this range.

Suppose the collective preferences of the owners can be represented by a utility function, say,

$$U(w) = -w^{-1}.$$

Table 3 shows the owners' terminal wealth as a function of the 'true price' which is revealed in the aftermarket.

The expected utility of the owners is, accordingly,

$$\begin{aligned} EU(w) &= p(\bar{v} > p)U(pZ) + p(\bar{v} < p) \\ &\quad \times E(U(NT(b, p) + \bar{v}(Z - NT(b, p)/p)) | \bar{v} < p). \end{aligned}$$

Under the assumption that  $\bar{v}$  is uniform on  $(0, 2\bar{v})$ , we have

$$EU(w) = \frac{-1}{pZ} + \frac{1}{2\bar{v}Z} \left\{ 1 + \frac{\log(NT(b, p)/pZ)}{1 - (NT(b, p)/pZ)} \right\}.$$

<sup>3</sup>For greater realism, we drop the fiction that the underwriting investment bank is an invisible intermediary. Henceforth, 'owners' refer either to the founders of the company or to the investment banking firm which acquires the shares prior to a public offering as part of a firm commitment arrangement.

Table 3

Issuer's terminal wealth, as a function of the realized value of the shares in the aftermarket.

	Aftermarket value <sup>a</sup>	
	$\bar{v} > p$ (underpriced)	$\bar{v} < p$ (overpriced)
Issuer's wealth	$pZ$	$NT(b, p) + \bar{v}(Z - NT(b, p)/p)$

<sup>a</sup>Aftermarket value is the price,  $v$ , realized on the first trade; this price differs from the offering price,  $p$ , according to whether the issue is underpriced ( $v > p$ ) or overpriced ( $v < p$ ). The number of shares offered is  $Z$ , the number of uninformed investors is  $N$ , and  $T$  is the fraction of wealth the uninformed investor allocates to the new issue.

This expression is easily interpreted. The first term,  $(-1/pZ)^{-1}$ , is the utility of the owners, given the issue is completely subscribed at the price  $p$ . The second term is a correction that depends upon the fraction of the issue subscribed by the uninformed:

$$NT(b, p)/pZ.$$

Observe that the issuer will not choose a price at which uninformed demand is zero. If the true value of the shares is lower than  $p$ , none of the shares are sold. The owners – either the investment bank which takes the shares public or the original organizers of the company – retain by default an issue which might be completely worthless. Given the form of the owners' utility function, the prospect of a total loss makes their expected utility equal to  $-\infty$ . Consequently, the owners are willing to offer the shares at a discount to enjoy the benefits of risk-sharing.<sup>4</sup>

Using the definition of  $b(p, N)$ , the probability of receiving an allocation, the maximization problem can be written

$$\max_{p > p_i} \frac{-1}{pZ} + \frac{1}{2\bar{v}Z} \left\{ 1 + \frac{\log\left(\frac{1}{b(p, N)} - \frac{\bar{v}}{p}\right)}{1 + \frac{\bar{v}}{p} - \frac{1}{b(p, N)}} \right\}.$$

<sup>4</sup>The fact that the owners are phobic about losing all their money does not mean they are unwilling to initiate such a project; they must simply expect to trade out before the ultimate outcome is known.

If the number of investors is large, then  $b(p, N)$  is approximately equal to the zero demand probability,  $b_0(p)$ . For the example considered in section 4, the zero demand probability is calculated explicitly. In this instance, the maximization problem becomes

$$\max_{p > 0.8\bar{v}} \frac{-1}{pZ} + \frac{1}{2\bar{v}Z} \left\{ 1 + \frac{p^2 \log((4\bar{v} - p)(\bar{v} - p)p^{-2})}{5\bar{v}(p - 0.8\bar{v})} \right\}.$$

The solution to the constrained maximization occurs at the boundary,  $0.8\bar{v}$ . The owners choose the smallest discount which guarantees full subscription in every state of the world.

## 7. Conclusion and empirical tests

The model presented here is designed to explain the well-established phenomenon of the underpricing of initial public offerings. Insofar as the underpricing feature is the focus of the model, it can hardly be considered the definitive test.

The crucial test of the model involves observing the degree to which shares are rationed on the offer date. If the model is correct, weighting the returns by the probabilities of obtaining an allocation should leave the uninformed investor earning the riskless rate.

Evidence on the degree of rationing, however, is hard to obtain for several reasons. First, underwriters are sensitive to the question of allocational fairness. They are generally hostile to the suggestion that shares are rationed in a preferential way, or, indeed, rationed at all. Second, the degree to which shares are rationed reflects directly on the underwriter's ability. If the shares are undersubscribed, the underwriter is either negligent in pricing them or lax in promoting and distributing the offer. If, however, the shares are heavily oversubscribed, the underwriter appears to be underpricing the issue to make it easier to sell. Finally, not all the orders the underwriter receives are firm. Orders are slightly inflated because the investor can renege on the deal for several days after the offer date. Hence, the degree of rationing is overstated.

Since direct evidence on the occurrence of rationing is hard to obtain, indirect evidence must be used instead. One indicator of the extent of rationing can be found in the sample of stocks which are offered with overallotment options. If, in this sample, the overallotment option is rarely exercised, then it is safe to say that rationing seldom occurs for initial public offerings as a whole. If, however, the overallotment provision is used frequently to obtain additional shares, then it seems that oversubscription is a common event. Of course, the mere existence of rationing is not sufficient to explain the discount. The important additional consideration is that rationing occurs more often for

good shares than for bad shares. To confirm the presence of a bias, those shares for which the issuer exercises the option must be shown to appreciate more in price than those shares for which the option expires unused. Such a test is easy to perform by simply measuring the price change of each class of shares on the offering date.

An alternative to measuring the extent of rationing in new issue markets is to find evidence of a similar effect in other markets. The same argument which predicts a discount for initial public offerings predicts a premium for tender offers. Briefly, uninformed investors expect the tender to be oversubscribed if the tender price is too high and undersubscribed if the tender price is too low. Thus, by participating in the offer, the uninformed give up their good shares and keep their bad shares. That is, the informed crowd the uninformed out of those tender offers for which the premium is too high and they withdraw from those tenders for which the premium is too low. To induce a sufficient number of uninformed investors to tender, therefore, the firm making the offer must add a premium to overcome the bias.

The advantage of using tender offers as a 'proof of principle' for initial public offerings is that the degree of rationing is a matter of public record. Firms announce, at the close of the offer, the number of tenders they receive. An additional advantage is that the allocation mechanism is explicit. In the case of oversubscription, the offers to tender are placed in a pool from which they are drawn in a prescribed way.<sup>5</sup> Finally, the premium is easy to measure; it is the simply ratio of the tender price to the *post-tender* trading price.

The model presented here for firm commitment offerings can be generalized considerably. Suppose that, instead of the orders *all* being received on one day and filled by lot, the orders arrive over a period of many days and are filled in order of arrival. Such an arrangement is typical of a 'best efforts underwriting'. If the issuer closes the offer as soon as the last share is subscribed, the rationing is 'invisible' because the unfilled orders can't be seen. Nevertheless, the unfilled orders exist; they belong to the disappointed buyers who arrive after all the shares are sold.

Invisible rationing exerts the same downward pressure on the offering price as the more overt kind. Uninformed investors who arrive in time suspect their success is due as much to lack of interest on the part of informed investors as it is to good luck. Conditional upon receiving an allocation, the uninformed find the shares to be worth less than their unconditional value. Therefore, just as in a firm commitment offering, the shares must be priced at a discount to attract uninformed buyers.

<sup>5</sup>In one type of tender offer [Federal Register (1984)]: 'The bidder states a maximum number of shares to be purchased in addition to a minimum condition. If the offer is oversubscribed, the tendered shares will be subject to prorationing. When the offer is executed under prorationing, each tendering account has the same fraction accepted. Prorationing requirements insure that each target shareholder receives proportionate share of the terms of the tender offer.'

Such an extension is satisfying because it suggests that the institutional mechanism for delivering the shares to the public is irrelevant as far as the offer price discount is concerned. Whether the shares are sold sequentially, as in the best efforts arrangement, or all at once, as in the firm commitment underwriting, the essentials are the same. The uninformed compete with the informed, and the issuer must ultimately compensate them for their disadvantage.

**Appendix**

First, we must prove some miscellaneous results.

*Lemma 1.*  $T(b, p)$  is strictly increasing in  $b$ , for  $p, b > 0$ .

*Proof.* Differentiate eq. (6) with respect to  $b$ :

$$\begin{aligned} & p(\tilde{v} > p)E[U'(1 + T(p^{-1}\tilde{v} - 1))(p^{-1}\tilde{v} - 1)|\tilde{v} > p] \\ & + (b)p(\tilde{v} > p)E[U''(1 + T(p^{-1}\tilde{v} - 1))(p^{-1}\tilde{v} - 1)^2|\tilde{v} > p] \frac{dT(b, p)}{db} \\ & + p(\tilde{v} < p)E[U''(1 + T(p^{-1}\tilde{v} - 1))(p^{-1}\tilde{v} - 1)^2|\tilde{v} < p] \frac{dT(b, p)}{db} = 0. \end{aligned}$$

Note that

$$U''(1 + T(p^{-1}\tilde{v} - 1))(p^{-1}\tilde{v} - 1)^2 < 0 \quad \text{if } U'' < 0.$$

Also

$$U'(1 + T(p^{-1}\tilde{v} - 1))(p^{-1}\tilde{v} - 1) > 0 \quad \text{when } \tilde{v} > p,$$

provided  $U' > 0$ . Thus, the first term above is positive while the coefficients of  $dT(b, p)/db$  are negative when  $b, p > 0$ . This implies

$$dT(b, p)/db > 0 \quad \text{for } p, b > 0.$$

proving the lemma.

*Lemma 2.*  $p \frac{d}{dp} \ln b_0(p) > 1$ .

*Proof.* Upon taking the logarithm of eq. (11), which defines  $b_0(p)$ , and differentiating,

$$\begin{aligned} \frac{d}{dp} \ln b_0(p) &= \frac{1}{p(\bar{v} < p)E(p - \bar{v}|\bar{v} < p)} \frac{d}{dp} E(p - \bar{v}|\bar{v} < p)p(\bar{v} < p) \\ &= \frac{1}{p(\bar{v} > p)E(p - \bar{v}|\bar{v} > p)} \frac{d}{dp} E(p - \bar{v}|\bar{v} > p)p(\bar{v} > p). \end{aligned}$$

Notice

$$\frac{d}{dp} E(p - \bar{v}|\bar{v} < p)p(\bar{v} < p) = p(\bar{v} < p),$$

and

$$\frac{d}{dp} E(\bar{v} - p|\bar{v} > p)p(\bar{v} > p) = p(\bar{v} > p).$$

Therefore,

$$p \frac{d}{dp} \ln b_0(p) = \frac{p}{E(p - \bar{v}|\bar{v} < p)} + \frac{p}{E(p - \bar{v}|\bar{v} > p)}.$$

Since  $\bar{v} > 0$ ,  $E(p - \bar{v}|\bar{v} < p) < p$ . Hence

$$p \frac{d}{dp} \ln b_0(p) > \frac{p}{p} = 1,$$

proving the lemma. We can now prove the following theorem.

*Theorem 1*

$$b = \frac{pZ}{NT^*(b, p) + I} \tag{11}$$

has the unique solution  $b(p, N) > b_0(p) > 0$ .

*Proof.* Write the tautology

$$b_0(p) = b_0(\bar{v}) \exp \left[ \int_{\bar{v}}^p \frac{d}{dt} \ln b_0(t) dt \right].$$

By Lemma 2 and assumption A.3,

$$b_0(p) < b_0(\bar{v}) \exp \left[ \int_{\bar{v}}^p t^{-1} dt \right] = b_0(\bar{v}) p \bar{v}^{-1} = \frac{pZ}{\bar{v}Z} < \frac{pZ}{I}.$$

Because  $T(b_0(p), p) = 0$  by definition, the inequality above implies

$$b_0(p) < \frac{pZ}{NT^*(b_0(p), p) + I} = \frac{pZ}{I}. \tag{12}$$

By Lemma 1,  $T(b, p)$  is strictly increasing in  $b$ . Therefore, for any  $b > (pZ/I)$ ,

$$b > \frac{pZ}{NT^*(b, p) + I}. \tag{13}$$

Both sides of expression (13) are continuous in  $b$ . By (12) and (13), the functions cross on  $(b_0(p), \infty)$ . Therefore, at some point,  $b(p, N)$ , which lies in the interval  $(b_0(p), \infty)$ , the functions are equal. Since the function on the left-hand side is strictly increasing, while the function on the right-hand side is decreasing, necessarily the solution  $b(p, N)$  is unique.

*Lemma 3.* Let  $(\partial^2/\partial b \partial p)T(b, p)$  and  $(\partial^2/\partial b^2)T(b, p)$  be continuous in the region  $0 < b < 1$ ,  $0 < p < \bar{v}$ . Then, for all  $p$  in any closed subinterval of  $(0, \bar{v})$  which does not contain 0, the following limits hold uniformly:

$$\lim_{N \rightarrow \infty} b(p, N) = b_0(p), \quad \lim_{N \rightarrow \infty} \frac{d}{dp} b(p, N) = \frac{d}{dp} b_0(p).$$

*Proof.* Differentiate eq. (11), which defines  $b(p, N)$ , with respect to  $N$ . Upon re-arranging,

$$\frac{d}{dN} b(p, N) = \frac{-pZ}{D^2} T(b(p, N), p) \left( 1 + \frac{pNZ}{D^2} T_b \right)^{-1},$$

where

$$D = NT(b(p, N), p) + I,$$

$$T_b = \frac{\partial}{\partial b} T(b(p, N), p).$$

By Lemma 1,  $T_b$  is positive, which implies

$$\frac{d}{dN} b(p, N) < 0 \quad \text{for } p > 0.$$

Since  $b(p, N)$  is decreasing in  $N$ , a limit exists which satisfies

$$\lim_{N \rightarrow \infty} b(p, N) = \lim_{N \rightarrow \infty} \left[ \frac{pZ}{NT(b(p, N), p) + I} \right].$$

From the above equation, either  $b(p, N)$  or  $T(b(p, N), p)$  goes to zero. The former possibility can be excluded by Theorem 1, for  $b(p, N) > b_0(p) > 0$ . Thus,

$$\lim_{N \rightarrow \infty} T(b(p, N), p) = T\left(\lim_{N \rightarrow \infty} b(p, N), p\right) = 0,$$

which means that  $b(p, N)$  converges to the zero demand probability. That is,

$$\lim_{N \rightarrow \infty} b(p, N) = b_0(p).$$

Since the convergence is monotone in  $N$ , and the limit is continuous, 'Dini's Theorem' yields that the approach is uniform on any closed subinterval of  $(0, \bar{v})$  which does not contain zero [see Dieudonné (1969)].

To show that the derivatives are uniformly convergent, differentiate eq. (11) with respect to  $p$ , and re-arrange,

$$\frac{d}{dp} b(p, N) = \frac{-T_p + (Z/b(p, N)N)}{T_b + (pZ/b^2(p, N)N)}, \quad p > 0,$$

where

$$T_p = \frac{\partial}{\partial p} T(b(p, N), p).$$

Observe that upon totally differentiating the identity  $T(b_0(p), p) = 0$ , we obtain

$$\frac{d}{dp} b_0(p) = - \frac{\partial}{\partial p} T(b_0(p), p) / \frac{\partial}{\partial b} T(b_0(p), p).$$

Compare the expressions for  $(d/dp)b(p, N)$  and  $(d/dp)b_0(p)$ . Since  $(\partial/\partial b)T(b_0(p), p) > 0$  for  $p > 0$  by Lemma 1, it suffices to show the following limits are uniform on any closed subinterval of  $(0, \bar{v})$  which does not contain zero:

$$\lim_{N \rightarrow \infty} \frac{\partial}{\partial p} T(b(p, N), p) = \frac{\partial}{\partial p} T(b_0(p), p),$$

$$\lim_{N \rightarrow \infty} \frac{\partial}{\partial b} T(b(p, N), p) = \frac{\partial}{\partial b} T(b_0(p), p).$$



By assumption,  $(\partial^2/\partial p \partial b)T(b, p)$  is continuous and bounded for  $0 < x \leq p \leq \bar{v}$ ,  $0 < b \leq 1$ . Let

$$M_x = \max_{\substack{0 < b < 1 \\ x < p < b}} \frac{\partial^2}{\partial p \partial b} T(b, p) < \infty.$$

By the Mean Value Theorem,

$$\frac{\partial}{\partial p} T(b(p, N), p) - \frac{\partial}{\partial p} T(b_0(p), p) < M_x b(p, N) - b_0(p).$$

Since  $b(p, N)$  converges uniformly to  $b_0(p)$  on  $0 < x \leq p \leq \bar{v}$ , then  $(\partial/\partial p)T(b(p, N), p)$  must converge uniformly to  $(\partial/\partial p)T(b_0(p), p)$ . An identical argument establishes that  $(\partial/\partial b)T(b_0(p), p)$  converges uniformly to  $(\partial/\partial b)T(b_0(p), p)$ , completing the proof of the lemma.

The uniform convergence of the function  $b(p, N)$  and its derivatives to  $b_0(p)$  is an important ingredient in the next theorem, which is the key to characterizing the issuer's opportunity set.

*Theorem 2.*  $(d/dp)T(b(p, N), p) < 0$ , for  $N$  sufficiently large.

*Proof.* Re-arrange eq. (11),

$$T(b(p, N), p) = \frac{pZ}{Nb(p, N)} - \frac{I}{N}.$$

This equation is an identity which can be differentiated with respect to  $p$ :

$$\frac{d}{dp} T(b(p, N), p) = \frac{Z}{b(p, N)N} 1 - p \frac{d}{dp} \ln b(p, N). \tag{14}$$

Since  $b(p, N)$  converges uniformly with its derivatives to  $b_0(p)$ , then

$$\lim_{N \rightarrow \infty} p \frac{d}{dp} \ln b(p, N) = p \frac{d}{dp} \ln b_0(p), \tag{15}$$

uniformly for all  $p$  in any closed subinterval of  $(0, \bar{v})$  which does not contain zero. By Lemma 2, the right-hand side of eq. (15) is greater than 1. Therefore, on any closed subinterval of  $(0, \bar{v})$  which excludes 0,  $N$  can be chosen large enough that the right-hand side of (14) is negative.

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