Yet Another ACD Model:

The Autoregressive Conditional Directional Duration (ACDD) Model

by

Nagaratnam Jeyasreedharan, University of Tasmania

David E Allen[1] Edith Cowan University and

Joey Wenling Yang, University of Western Australia

April 2014

Abstract

This paper features a new ACD model which sits within the theoretical framework provided by the recently developed observation-driven time series models by Creal, et al (2013): the Generalized Autoregressive Score (GAS) models. The ACDD model itself contains three novelties. First, durations (intra-trade intervals or waiting-times) are signed, based on whether a (positive) ask-driven trade or a (negative) bid-driven trade occurred. These signed trade-durations are known as directional durations. Second, as the resultant directional durations are no longer positive and asymmetrical but are symmetrically distributed, the familiar GARCH-like formulation of the ACD process is proposed for modelling these directional durations. Consequently, the proposed model is called the Autoregressive Conditional Directional Duration (ACDD) model. Third, using the alternative GARCH-like formulation, persistence or long-memory in the durations is easily addressed both via the mean and variance equations: the mean equation uses a Semi-parametric Fractional Autoregressive (SEMIFAR) formulation and the variance equation uses a

[1] Corresponding Author D.E. Allen, School of Accounting, Finance and Economics, Edith Cowan University, Joondalup, Western Australia 6027. Tel:61863045471. Fax: 6163045271. Email: d.allen@ecu.edu.au

GARCH formulation. The paper demonstrates the flexibility and convenience of the GAS model framework in the context of a particular ACD model specification. The model can be viewed as an alternative extension of the ‘Asymmetric ACD model’ of Bauwens and Giot (2003) which captures information related to the evolution of prices as well as the quote-durations.

Keywords: ACD model, ACDD model, directional duration, SEMIFAR, GAS models.

1 Introduction

High-frequency financial time series have become widely available during the past decade or so. Records of all transactions and quoted prices are readily available in pre-determined formats from many stock exchanges. An inherent feature is that such data are irregularly spaced in time. Several approaches have been taken to address this feature of the data.

The seminal work originated with Engle and Russell [9], where the time between events (trades, quotes, price changes etc.) or durations are the quantities being modelled. These authors proposed a class of models called the Autoregressive Conditional Duration, or ACD, models, where conditional (expected) durations are modelled in a fashion similar to the way conditional variances are modelled using ARCH and GARCH models of Engle [8] and Bollerslev [5].

ACD models and GARCH models share several common features, ACD models being commonly viewed as the counterpart of GARCH models for duration data. Both models rely on a similar economic motivation following from the clustering of news and financial events in the markets. The autoregressive ACD model captures the duration clustering observed in high frequency data, i.e. small (large) durations being followed by other small (large) durations in a way similar to the way the GARCH model accounts for volatility clustering. Just as a low order GARCH model is often found to suffice for removing the dependence in squared returns, a low order ACD model is often successful in removing the temporal dependence in durations (see Pacurar [16]). Following the GARCH literature, a number of extensions to the original linear ACD model by Engle and Russell [9] have been suggested. These include the logarithmic ACD model of Bauwens and Giot [1], and the threshold ACD model of Zhang, Russell and Tsay [17]. The error distributions associated with the conditional durations has also been suggested to have several different shapes. Examples include the exponential and Weibull distributions as in Engle and Russell [9], and the Burr and generalized gamma distributions utilised by Grammig and Maurer [9] respectively. However, a crucial assumption for obtaining the quasi-maximum likelihood (QML) consistent estimates of the ACD model and its extensions is that the conditional expectation of durations is correctly specified and that the model is linear. The QML estimations yield consistent estimates and the inference procedures in this case are straightforward to implement, but this comes at the cost of efficiency. *In practice, fully efficient maximum likelihood (ML) estimates might be preferred* if the nature of the underlying distribution is known; however, this is not likely to be the case.

The original ACD models focus on taking into account the duration between market events; quote or price changes, and did not include information inherent in the evolution of the price process in the dynamics of the model. A significant departure from this is the Asymmetric ACD model of Bauwens and Giot [2] who follow a direction first explored by Russell and Engle (2002) in their Autoregressive Conditional Multinomial Model which featured an ACD model fitted to the durations plus a generalized linear model of the conditional transition probabilities of the price process. The advantage of this type of approach is that other market-microstructure related information such as the traded volume and the corresponding transaction prices, bid and ask quotes offered by the market makers, can be directly included to enhance the precision and forecasting ability of the model.

The model developed in this paper is a variant of the asymmetric approach explored by Bauwens and Giot [2] and it sits within the context of recent work by Creal, et al [6] in their development of the Generalized Autoregressive Score (GAS) models provides a natural framework for our model. This new class of observation-driven time series models adopts a mechanism to update the parameters over time by using the scaled score of the likelihood function. This approach provides a unified and consistent framework for introducing time-varying parameters in a wide class of nonlinear models. They suggest that their GAS model encompasses other well-known models such as the generalized autoregressive conditional heteroskedasticity (GARCH models), autoregressive conditional duration (ACD models), autoregressive conditional intensity, and Poisson count models with time-varying means.

Time series models with time-varying parameters can be divided into two classes of models: observation-driven models and parameter-driven models. In the former approach, time variation of the parameters is introduced by letting parameters be functions of lagged dependent variables as well as contemporaneous and lagged exogenous variables. Although the parameters are stochastic, they are perfectly predictable given the past information. This simplifies likelihood evaluation and observation-driven models have become popular in the applied statistics and econometrics literature. Typical examples of these models are the generalized autoregressive conditional heteroskedasticity (GARCH) models of Engle [8], and Bollerslev [5], and the autoregressive conditional duration (ACD) and model of Engle and Russell [9]. In the latter, parameter-driven models, the parameters are stochastic processes with their own sources of error. An example of this class of models would be stochastic volatility models, as discussed by Shephard [17].

Creal, et al [6] formulate their general class of observation-driven time-varying parameter models and exploit the full density structure of the score function. In this class of models, the time-varying parameter f_t and the score depend on the full underlying density structure. They demonstrate that their GAS model structure can nest both GARCH (1,1) models and ACD (1,1) models as well as MEM models (multiplicative error models).

They proceed as follows: Let $Nx1$ vector y_t represent the dependent variable of interest, f_t the time varying parameter vector, x_t a vector of exogenous variables, (covariates), all at time t, and θ a vector of static parameters. Define $Y^t = \{y_1, \ldots, y_t\}$, $F^t = \{f_0, f_1, \ldots, f_t\}$ and $X^t = \{x_1, \ldots x_t\}$. The available information set available at time t consists of $\{f_t, F^t\}$, where

$$\mathrm{F}_t = \{Y^{t-1}, F^{t-1}, X^t\}, \text{ for } t = 1, \ldots, n$$

It is assumed that y_t is generated by the observation density

$$y_t \sim p(y_t | f_t, \mathrm{F}_t; \theta) \tag{1}$$

To set the model framework in the familiar autoregressive context that provides the context for both GARCH and ACD models assume that the mechanism for updating the time-varying parameter f_t is given by an autoregressive updating equation:

$$f_{t+1} = \omega + \sum_{i=1}^{p} A_i s_{t-i+1} + \sum_{j=1}^{q} B_j f_{t-j+1} \qquad (2)$$

Where ω is a vector of constants, the coefficient matrices $\boldsymbol{A}_i$ and $\boldsymbol{B}_j$ have the appropriate dimensions for $i = 1, \ldots, p$ and $j = 1, \ldots, q$, while s_t is an appropriate function of past data. The unknown coefficients to be estimated in the expression above are functions of θ. Clearly, both GARCH and ACD models sit within this general GAS framework.

Our model developed in this paper presents a simple modification of the basic ACD model. The inherent limitations in the ACD model and its extensions to date have been a direct consequence of the positive asymmetric density assumed for the innovations, ε_i in all these models; as time between successive trades are positive (see Hautsch [13]). Distributions defined on positive support typically imply a strict relationship between the first moment and higher order moments and do not disentangle the conditional mean and variance function. For example, under the exponential distribution, all higher order moments directly depend on the first moment. Consequently, the corollary as derived in Engle and Russell [9] using the EACD(1,1) model cannot not necessarily be extended to the more general ADCD(p,q) models with further proofs (see Pacurar [16]). Hence there is a certain inflexibility and lack of published rigorous diagnostics encountered with standard

ACD models. Explicit GARCH-based ACD models circumvent these limitations for obvious reasons.

In addition, apart from being autocorrelated and having arch effects, duration innovations also exhibit long range dependence (long memory) and non-stationarity. Empirical studies based on the linear ACD model often reveal persistence in durations as the estimated coefficients on lagged variables add up nearly to one. Moreover, many financial duration series show a hyperbolic decay, i.e. significant autocorrelations up to long lags. This suggests that a better fit might be obtained by accounting for longer term dependence in durations. Indeed, the standard ACD model imposes an exponential decay pattern on the autocorrelation function typical for stationary and invertible ARMA processes. This may be completely inappropriate in the presence of long memory processes. Thus, whilst crucial for the ACD model and its extensions the "assumptions of iid innovations may be too strong and inappropriate for describing the behaviour of trade durations" (see Pacurar [16]). A further point of note is that whilst the Ljung-Box test statistic is assumed to have an asymptotic χ^2 distribution under the null hypothesis, no formal analysis exists that rigorously establishes this result in the context of the standard ACD models (see Pacurar [16]).

In this paper we provide a slightly different approach to work originated by Engle and Russell [9]. We propose an alternative definition of durations, where positive durations depict "ask-durations" and negative durations depict "bid-durations". This approach enables the innovation error density to be symmetrical. The ensuring model is called the Autoregressive Conditional Directional Duration (ACDD) model.

Bid and ask durations can be important individual conveyors of market microstructure information (see Bauwens and Giot [2], Easley and O'Hara [7]). Zhang et al [19] demonstrate that the decomposition of the spread into two components: the cost of buy exposure and the cost of sell exposure by taking into account the time series characteristics of trading at the bid and ask produces richer information about trading costs and price volatility. They test and find evidence that the effect of volumes traded on these components is not symmetric, which is an effect not captured in standard ACD models which do not distinguish between trading at the bid and ask. Our model framework would facilitate the greater exploration of these effects if warranted.

A further consideration is that recently there have been considerable advances in algorithmic trading and in market surveillance techniques utilized by regulators. They both utilize the analysis of microstructure patterns of buying and selling sequences. If any patterns are found to be extractable, they will be invaluable for smart traders. Other distinct microstructure patterns may reflect abnormal trading behaviour by market participants. These microstructure patterns can then be used to empower market trading/surveillance agents in monitoring the markets

The paper is organized as follows; we have set the scene in the introduction and briefly introduced GAS models which provide a broad conceptual framework for a wide variety of GARCH and ACD models. In section two we briefly discuss the standard ACD model and introduce the concept of directional durations. Section three introduces the semi-parametric fractional autoregressive ACDD model, and the research method and data are discussed in sections four and five. The results are discussed in section six and section seven concludes the paper.

2 The basic ACDD model

The time series of arrival times or durations between successive occurrences of certain events associated with the trading process can be defined in a number of ways. Examples include the time between successive trades, the time until a price change occurs or until a pre-specified number of shares or level of turnover has been traded. We define directional durations as signed durations or times between successive trades. The signs of the durations are positive when the trade price is above the mid-price and are negative when the trade price is below the mid-price. The sign of the duration when the trade-price is equal to the mid-price (13.25% of the data) is replaced with the directional sign of the previous directional duration. The mid-price is taken to be the average of the nearest bid and ask quotes. In doing so, we are able to differentiate between the arrival times of bid and ask-driven trades.

The basic ACDD model relies on a linear parameterization of the conditional duration, Ψ_i which depends on *p* past absolute directional durations and *q* past conditional durations, defined as:

$$\Psi_i = \omega + \sum_{j=1}^{p} \alpha_j \left|\delta_{i-j}\right| + \sum_{j=1}^{q} \beta_j \Psi_{i-j} \tag{3}$$

where $\delta_i = \gamma_i (t_i - t_{i-1})$ are the directional durations and γ_i = *1* for ask durations i.e. when the trade price is greater than the mid-price and γ_i = *-1* for bid durations i.e.

when the trade price is lower than the mid-price, with *t* being the trade times. To ensure positive conditional durations for all possible realisations, sufficient but not necessary conditions are $\omega > 0, \sum_{j=1}^{p} \alpha_j \geq 0, \sum_{j=1}^{q} \beta_j \geq 0$. The main assumption behind ACDD model is that the standardised directional durations,

$$\varepsilon_i = \frac{\delta_i}{\psi_i}, \tag{4}$$

are independent and identically distributed (IID) with $E(\varepsilon_i) = 0$ and $E(\varepsilon_i^2) = 1$[2]. Equation (3) is analogous the standard ACD model with the exception of directional durations, $\delta_i = \gamma_i(t_i - t_{i-1})$ as defined above. The significance of $|\delta_{i-j}|$ in the ACDD model to is to ensure non-negative durations in the conditional duration process.

A natural choice convenient for estimation could be any family of suitable symmetrical distributions. We adopt the generalized error distribution (GED) family proposed by Nelson [15] to capture the fat tails, if any, in the error terms. Let $f(\varepsilon, \theta_\varepsilon)$ be the density function for ε with parameters θ_ε. If a random variable, ε_i has a GED with mean zero and unit variance, the PDF of ε_i is given by:

$$f(\varepsilon_i) = \frac{\upsilon \exp\left[-(\frac{1}{2}) \left| \varepsilon_i / \lambda \right|^{\upsilon} \right]}{\lambda \bullet 2^{(\upsilon+1)/\upsilon} \Gamma(1/\upsilon)} \tag{5}$$

where

$$\lambda = \left[\frac{2^{-2/\upsilon} \Gamma(1/\upsilon)}{\Gamma(3/\upsilon)} \right]^{1/2} \tag{6}$$

---

[2] Note that the standard ACD model assumes the standardised durations are independent and identically distributed (IID) with $E(\varepsilon_i) = 1$ and $E(\varepsilon_i^2) = 2$.

and υ is a positive parameter governing the thickness of the tail behaviour of the distribution. When υ=2 the above PDF reduces to the standard normal PDF; when υ<2, the density has thicker tails than the normal density; when υ>2, the density has thinner tails than the normal density. When the tail thickness parameter υ=1, the PDF of the GED reduces to the PDF of a double exponential distribution (The GED nests the Exponential pdf distribution in the basic ACD model of Engle and Russell [9]).

Based on the above PDF, the log-likelihood function of ACDD model with GED errors can be constructed and maximum likelihood (ML) and quasi-maximum likelihood (QML) estimators for the ACDD parameters can be easily derived.. Furthermore, the redefinition of durations to bid- and ask-based durations enables us to fully adopt the full range of extant GARCH formulations i.e. meaning both the mean equation and the variance equation in the standard GARCH model and its various extensions can be utilised for duration modeling. Various types of GARCH models, such as EGARCH, TGARCH, PGARCH, etc. can be accessed for analogous ACDD modelling but will not be considered here as the motivation in this paper is to highlight and investigate the effects of embedding the bid-ask trading dynamics into the duration processes against the standard ACD approach used in Engle and Russell [9]. Investigations into the relevance of the other GARCH types for ACD modeling (including nonlinear models) are left for future research.

Under the proposed ACDD formulation, the directional durations are open to long range dependence (long memory) and non-stationarity, if any, in addition to exhibiting autocorrelation, arch and diurnal effects (see Table 2). To address these additional stylised characteristics and as several ‘trend-generating’ mechanisms may be occur simultaneously, we introduce a SEMIFAR-based mean equation into the ACDD model.

3 The SEMIFAR-ACDD model

Semi-parametric fractional autoregressive (SEMIFAR) models (see Beran and Feng [3], [4]) have been introduced for modelling different components in the mean function of a financial time series simultaneously, such as nonparametric trends, stochastic nonstationarity, short- and long-range dependence as well as anti-persistence. SEMIFAR includes ARIMA and FARIMA processes (see Hosking [14]; Granger and Joyeux [12]).

Let $d = (-0.5, 0\ .5\ )$ be the fractional differencing parameter, $m \in (0,1)$ be the integer differencing parameter, L be the lag or backshift operator, $\phi\ (\ L\ )$ be the lag polynomials in L with no common factors and all roots outside the unit circle and ε_i be white noise, then the SEMIFAR model can be defined as (see Feng, Beran and Yu [10]):

$$\phi(L)(1-L)^d\left[(1-L)^m y_i - g(\tau_i)\right] = \varepsilon_i \tag{7}$$

where $\tau_i = t_i / n$

Similarly, in the SEMIFAR–ACDD model, the mean equation is defined as follows:

$$\phi(L)(1-L)^d\left[(1-L)^m \delta_i - g(\tau_i)\right] = \varsigma_i \tag{8}$$

with the duration equation defined by:

$$\psi_i = \omega + \sum_{j=1}^{p} \alpha_j \left|\varsigma_{i-j}\right| + \sum_{j=1}^{q} \beta_j \psi_{i-j} \tag{9}$$

where ς_i is then the SEMIFAR-filtered directional duration. To ensure positive conditional durations for all possible realizations, sufficient but not necessary conditions are that $\omega > 0, \sum_{j=1}^{p} \alpha_j \geq 0, \sum_{j=1}^{q} \beta_j \geq 0$. The main assumption behind SEMIFAR-ACDD model is that the standardised directional durations,

$$\varepsilon_i = \frac{\varsigma_i}{\psi_i} \tag{10}$$

are independent and identically distributed (IID) with $E(\varepsilon_i) = 0$ and $E(\varepsilon_i^2) = 1$.

4 Methodology

Based on the (Semi-parametric Fractional Autoregressive) SEMIFAR-ACDD model above and the asymptotic results for the SEMIFAR-GARCH formulation obtained by Feng, Beran and Yu [10], the following algorithm in S-PLUS is proposed for the practical implementation of the SEMIFAR–ACDD model:

(a) Carry out data-driven SEMIFAR fitting using algorithm AlgB defined in Beran and Feng [3] on the square-root of directional durations to obtain $g(\tau)$ and $\phi(L)$;

(b) Calculate the residuals $\xi_i = \delta_i - g(\tau_i)$ and invert ξ_i using $\phi(L)$ into $\hat{\varsigma}_i$, the estimates of ς_i;

(c) Estimate the variance equation in ACDD model using S-PLUS/GARCH subroutine on the estimated residuals $\hat{\varsigma}_i$ of the SEMIFAR model from (b) above.

The best SEMIFAR-ACDD model is then determined as follows:

(a) For $p=1,p_{max}$ and $q=1,q_{max}$ estimate ACDD(p,q) and calculate it's Bayesian Information Criterion i.e. BIC(p,q);

(b) Choose the ACDD(p,q) model that minimizes the BIC. We obtain the best-fit ACDD model, using the BIC as defined by:

$$BIC(p,q) = -2 \bullet \log(\text{maximized likelihood}) + (\log n)(p+q+2) \tag{11}$$

With the trend function in the SEMIFAR–ACDD model, it is inconvenient to select the two equations (8 and 9) at the same time. As the estimated parameter vectors for the SEMIFAR and the ACDD models are asymptotically independent (see Feng, Beran and Yu [10]) we adopt a two-stage approach. The best-fit SEMIFAR(r) model is chosen from *r=0,1,2* and the best fit ACDD(p,q) model selected from *p=0,1,2* and *q=0,1,2* , via the AIC/BIC/LL scores.

5 The Data

The dataset used in this paper is the IBM data used in the seminal paper titled "Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data" by Engle and Russell [9] and was downloaded from *http://weber.ucsd.edu/~mbacci/engle*. This is to enable direct comparisons to be made with the standard ACD model using the same data. Engle and Russell [9] give the following account of the data set: "The data were abstracted from the Trades, Orders Reports, and Quotes (TORQ) data set constructed by Joel Hasbrouck and NYSE. The data set contains detailed information about each transaction occurring on the consolidated market during regular trading hours over a 3 month period beginning November 1, 1990 and ending January 31, 1991. In addition to information about bid and ask quote movements, the volume associated with the transactions, and the transaction prices, there is a time stamp, measured in seconds after midnight, reflecting the time at which the transaction occurred". A plot of the trade and quote transaction data is shown in Figure 1 below:

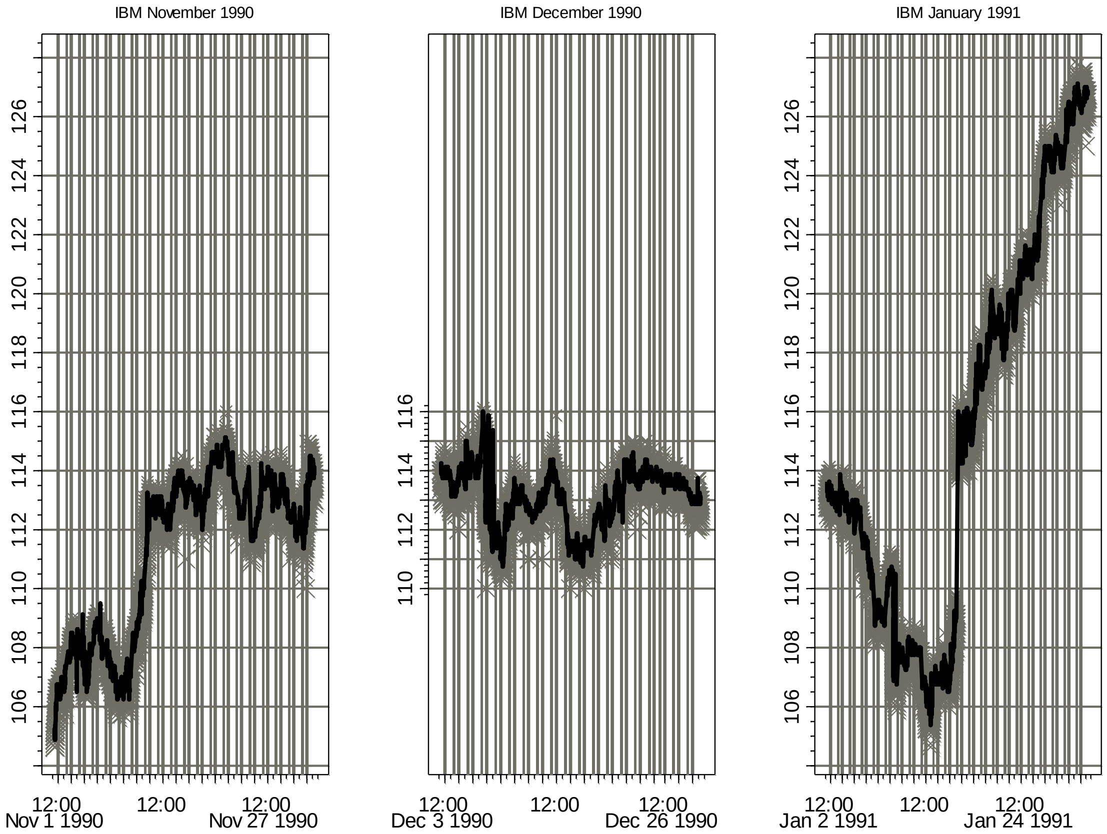


Figure 1: IBM transaction data by Engle and Russell [9]
Note: This is the original IBM data used by Engle and Russell [9]. It includes transactions from November 1990 through January 1991. The grey crosses depict the bid and ask quotes and the black line depicts the trade prices.

A total of 60328 transactions were recorded for IBM over the 3 months of trading on the consolidated market from November 1990 through January 1991. As per the seminal paper, 2 days from the three months of quote and trade data were deleted. A halt occurred on 23rd November and a more than one hour opening delay occurred on 27th December. Following Engle and Russell [9] the first half hour of the trading day (i.e. trades and quotes before 10.00am) is omitted. This is to avoid modelling the opening of the market which is characterized by a call auction followed by heavy trading activity as the dynamics are likely to be quite different over this call period. Furthermore, the call auction transactions are not recorded at the same time each morning.

In addition, all trades and quotes after 4.00pm were also omitted. After omitting these two days and deleting those trade times less than 10am and greater the 4pm, 51356 observations of the original 60328 transactions remained. Of the transactions occurring at non-unique trading times, nearly all of them corresponded with zero price movements. Engle and Russell [9] suggest that these transactions may reflect large orders that were broken up into smaller pieces. As it is not clear that each piece should be considered a separate transaction, the zero-second durations were considered to be a single transaction and were deleted from the data set. After all the adjustments to the data, 46052 observations were collated.

In their seminal paper, Engle and Russell [9] reported 46091 final IBM observations. This is probably a typo (it should have been 46051) as their other reported summary statistics for the same dataset was identical with the mean duration of 28.38 seconds, maximum duration of 561 seconds and standard deviation of 38.41 seconds obtained from out final dataset. We ended up with 46052 observations, the extra 1 observation is due to the way we adjusted our durations.

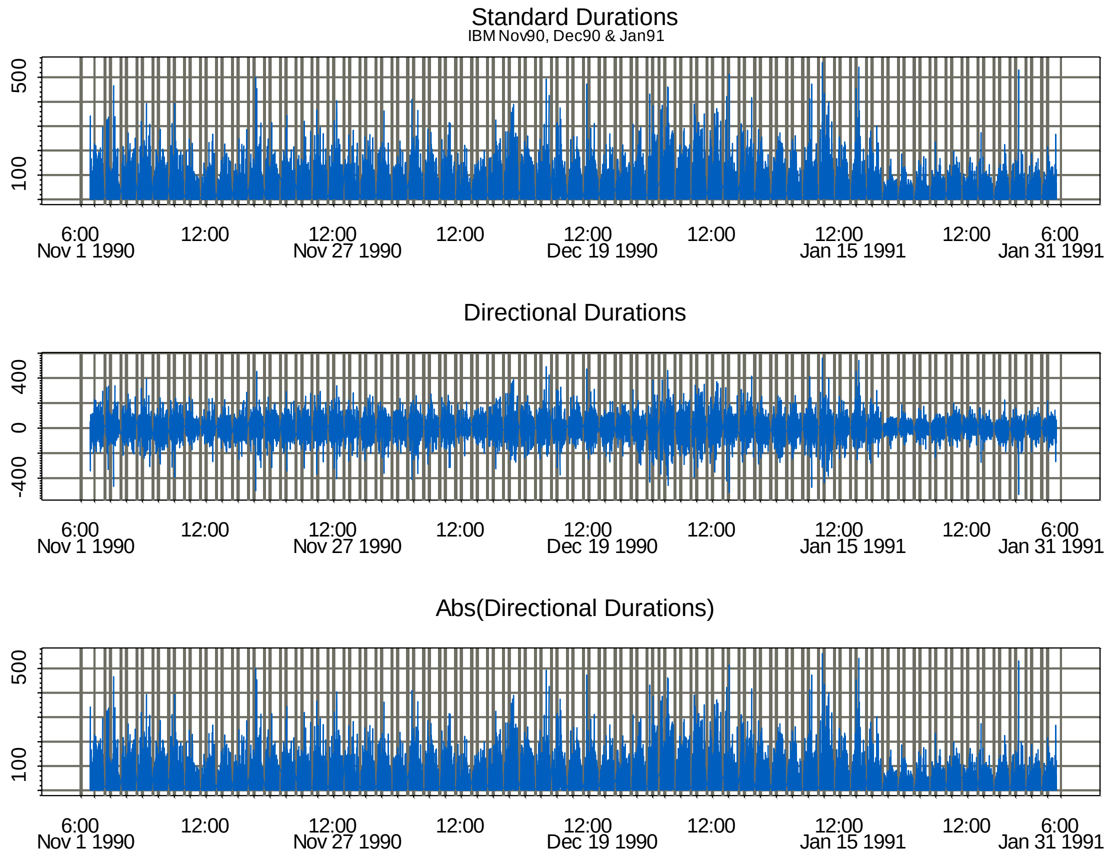


Figure 2: Standard, Directional and Abs(Directional Durations) in seconds
Note: As extracted from the original IBM data used by Engle and Russell [9]. It includes durations from November 1990 through January 1991.

In Figure 2 it can be seen that the directional durations can either be positive or negative, whereas standard durations have strictly positive support. In addition, absolute values of the directional durations are equivalent to standard durations. The directional durations as defined enable symmetrically distributed innovation errors to be assumed.

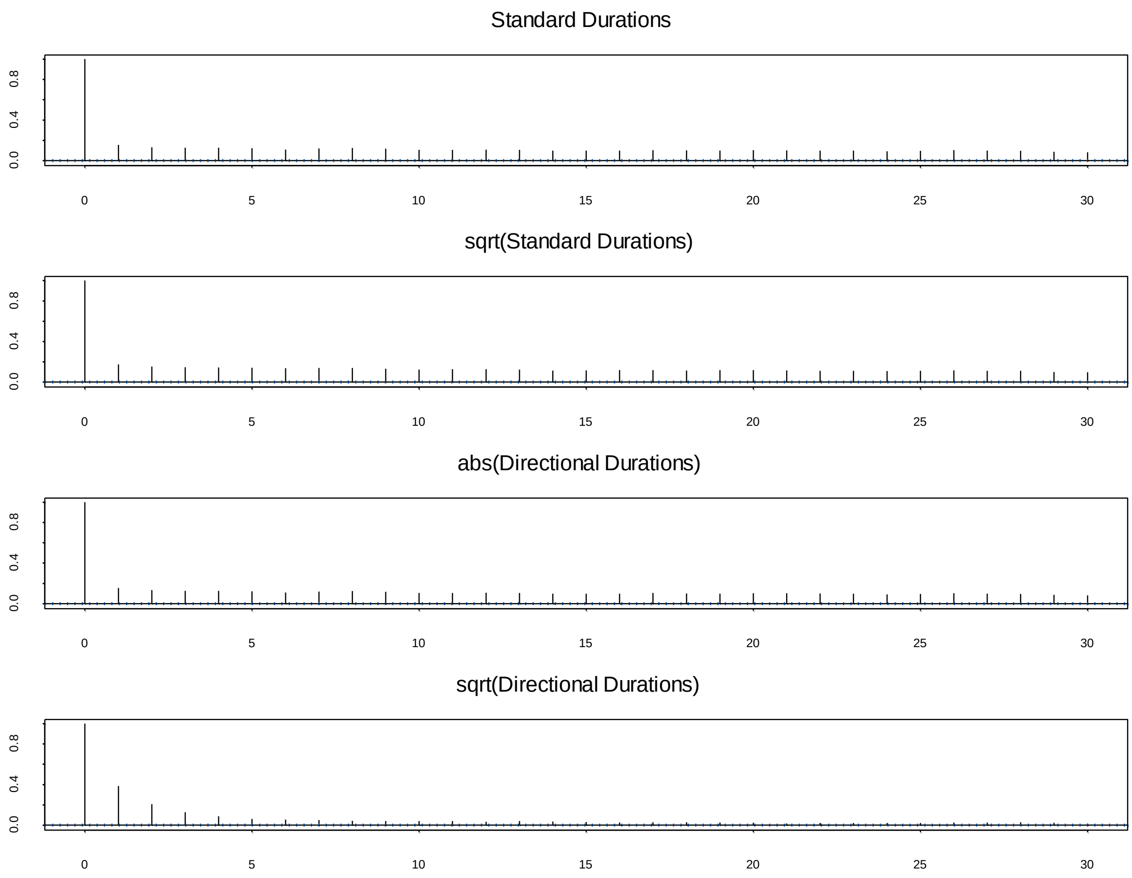


Figure 3: ACF plots of Standard and Directional Durations

It can be seen from Figure 3 that the autocorrelation properties of the standard duration and the absolute directional durations are identical. However, the ACF plot of the directional durations exhibit strong first-order AR(MA) behaviour. Herein the difference between standard and directional durations: the first order dependencies are fundamentally different. The inclusion of the SEMIFAR equation as the mean equation to ACDD model ensures that the first-order dependencies are addressed. However, the second order dependencies are identical. Consequently, the variance equation is analogous to that in the standard ACD model. Hence, the mean equation captures the additional information content embedded in directional durations. In this respect alone, the ACDD model can be deemed to be a more adequate model than the ACD for modelling (directional) durations data.

Descriptive statistics summarizing the characteristics of unadjusted standard, directional and absolute directional durations are shown in table 1 below:

	min	X1Q	median	X3Q	max	mean	std	skewness	kurtosis
SD	1	5	15	36	561	28.3845	38.4121	3.5467	23.6688
DD	-531	-12	3	18	561	3.0724	47.6629	0.0943	15.8655
abs(DD)	1	5	15	36	561	28.3845	38.4121	3.5467	23.6688

Table 1: Unadjusted Duration Descriptive Statistics
Note: SD (Standard Durations), DD (Directional Durations) and abs(DD) (Absolute Directional Durations).

It can be seen in Table 1 that using directional durations increases the range of durations, reduces their mean value, and reduces their skewness and kurtosis whilst adding to their standard deviation.

6 Results

The seasonal adjustment to the standard durations was carried out as done by Engle and Russell [9] using the same scatterplot smoothing SUPSMU-subroutine in S-PLUS. A SEMIFAR filter (the mean equation) was then applied both to the square-root adjusted standard durations and the square-root adjusted directional deviations.[3] This also enabled an equivalent SEMIFAR-ACD model to be compared against a similar SEMIFAR-ACDD model (as recommended by an anonymous referee).

[3] The transformation was carried out so that the ACD model can be estimated with GARCH software as per Engle and Russell[9].

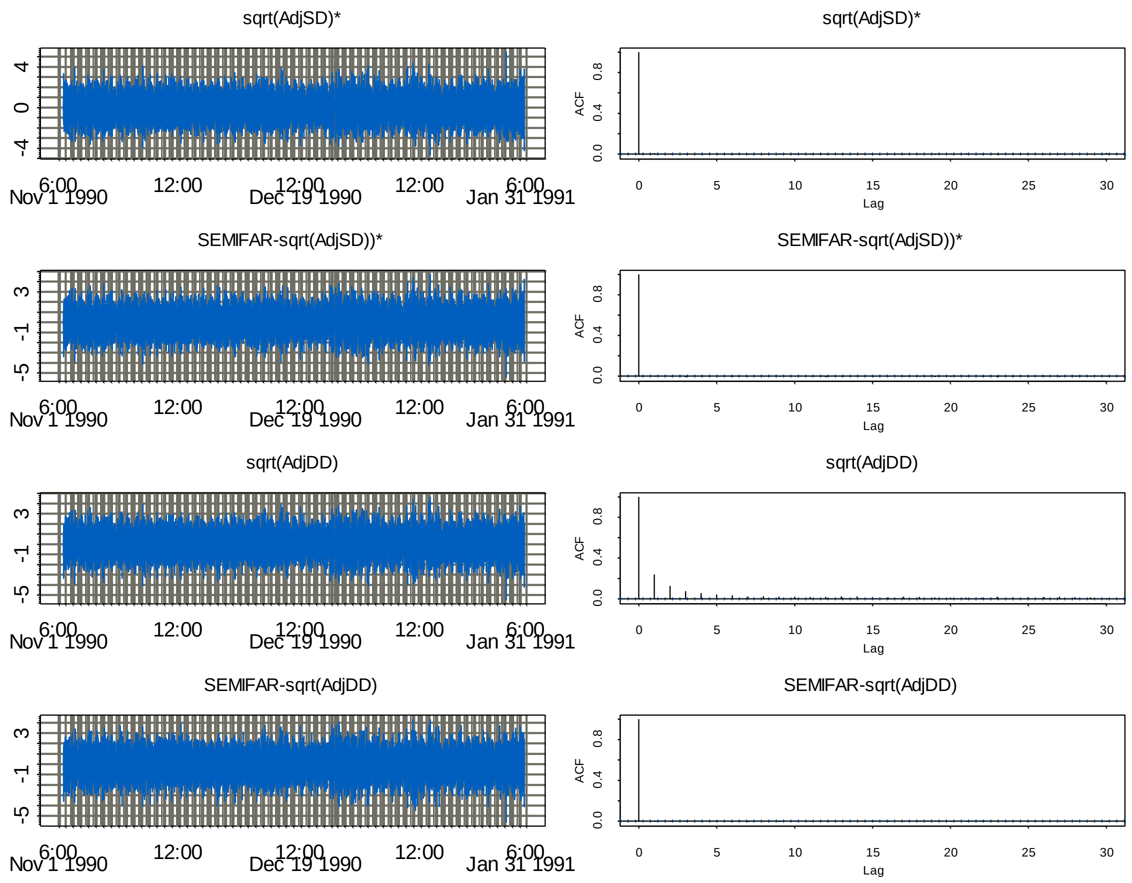


Figure 4: Square-Root Adjusted Durations and ACF plots
Note: *~The adjusted standard durations have been further transformed by a random series of -1s and +1s as per Engle and Russell [9].

Figure 4 exhibits the adjusted square-root durations (sqrt(Adj)) for both the standard and directional durations. As mentioned earlier, the algorithm B (AlgB) in Beran and Feng [3] was used for estimating the SEMIFAR portion of the model. The trend was estimated by local linear regression using a kernel as the weight function. For the short memory effects, only an AR (auto-regressive) component was considered. The SEMIFAR model is chosen from r=0,1,2. The optimal lag length obtained for the autoregressive portion of the SEMIFAR-ACD model was r=0 whereas the optimal lag length obtained for the autoregressive portion of the SEMIFAR-ACDD model was r=2. The LB (Ljung–Box test) -statistics, LM (Lagrange-Multiplier) -statistics, RS (Rescaled Range) -statistics and KPSS (Kwiatkowski–Phillips–Schmidt–Shin) -

statistics are listed in Table 2 for the adjusted and square-root adjusted standard and directional durations (AdjSD, AdjDD, sqrt(AdjSD) and sqrt(AdjDD)) before and after applying the SEMIFAR(2) filter.

	LB-stat	LM-stat	RS-stat	KPSS-stat
sqrt(AdjSD)*	45.3369	2828.9225	0.8276	0.0268
p.value	0.4999	0.0000	>0.10	>0.10
sqrt(AdjDD)	4345.0041	2828.9225	2.5465	0.549
p.value	0.0000	0.0000	<0.01	<0.05
SF-sqrt(AdjSD)*	68.7266	2825.3575	0.3398	0.0025
p.value	0.0166	0.0000	>0.10	>0.10
SF-sqrt(AdjDD)	32.831	2142.5727	0.9341	0.0315
p.value	0.9279	0.0000	>0.10	>0.10

Table 2: Adjusted and SEMIFAR-Adjusted Duration Statistics

Note: *~The adjusted standard durations have been further transformed by a random series of -1s and +1s as per Engle and Russell [9]. LB (Ljung–Box test), LM (Lagrange-Multiplier), RS (Rescaled Range), KPSS ((Kwiatkowski–Phillips–Schmidt–Shin).

The square-root adjusted standard durations (sqrt(AdjSD)) contain significant arch effects. The square-root adjusted directional durations (sqrt(AdjDD)) contain significant serial correlations, long memory and arch effects as expected. The SF(0)-sqrt(AdjSD) residual standard durations exhibit serial correlations and arch effects. However, all the SF(2)-sqrt(AdjDD) residual directional duration statistics are insignificant with the exception of significant arch effects, indicating the SEMIFAR filter has been efficient in capturing the serial correlation, long memory and non-stationarity in the adjusted directional durations. Consequently, the GARCH equation need only address the arch effects.

The ACD, SEMIFAR-ACD, ACDD and SEMIFAR-ACDD models are then fitted, diagnosed and compared. The model parameters for the mean and variance equations are listed in Table 3. The mean equation estimates are the same for both the ACD and ACDD models are null by construction. The ACD and ACDD variance equation parameter estimates are of the same order and significant (not shown). The fractional differencing parameter d estimate of 0.1017 for the SF(2)-ACDD(1,1) model indicates persistence in the bid-ask process is being captured by the mean equation.[4] The GED parameter estimates are greater than the value of 2 (for a normal distribution) for all standardized model residuals models. However, the GED distribtution for the SF(2)-ACDD(1,1) standardised residuals is 2.18, indicating a closer fit to a normal distribution.

	ACD(1,1)	SF(0)-ACD(1,1)	ACDD(1,1)	SF(2)-ACDD(1,1)
d	NA	-0.0095	NA	0.1017
AR(1)	NA	NA	NA	0.1135
AR(2)	NA	NA	NA	0.0287
A0	0.0111	0.0106	0.0102	0.0105
ARCH(1)	0.0635	0.0619	0.0617	0.0568
GARCH(1)	0.9262	0.9275	0.9282	0.9337
GED-v	2.9772	2.9336	2.9384	2.1808

Table 3: ACD, SEMIFAR-ACD, ACDD and SEMIFAR-ACDD Parameters

The descriptive and diagnostic statistics for the standardised residuals of various models are displayed in Table 4. The Ljung-Box statistic for the ACD(1,1), SF(0)-ACD(1,1) and SF(2)-ACDD(1,1) are all insignificant. However, the LB-statistic is 4053.15 for the ACDD(1,1) model, indicating high first-order dependency in the directional durations. This high dependency is subsequently completely addressed by the extended SEMIFAR(2)-ACDD(1,1) model with some nominal but statistically significant arch effects still remaining.

[4] The integer differencing parameter m is 0 for all SEMIFAR fits.

	mean	stdev	LB-stat	LM-stat	JB-stat
ACD(1,1)	-0.0023	1.0335	7.9554	20.6874	689.07
p.value	NA	NA	0.7886	0.0552	0.0000
SF(0)-ACD(1,1)	0	1.0332	9.3075	20.4316	688.4707
p.value	NA	NA	0.6765	0.0593	0.0000
ACDD(1,1)	0.1045	1.0282	4053.1534	20.8674	773.3262
p.value	NA	NA	0.0000	0.0524	0.0000
SF(2)-ACDD(1,1)	0	0.9937	6.3267	26.5903	251.478
p.value	NA	NA	0.8987	0.0088	0.0000

Table 4: ACD/ACDD and SF-ACD/ACDD Statistics on Std Residuals

To remove the remaining arch effects, a best-fit SEMIFAR(2)-ACDD model is selected from *p=0,1,2* and *q=0,1,2* by means of BIC. The best fitted ACDD model is found to be the SEMIFAR(2)-ACDD(2,1) as depicted by the lowest AIC/BIC/LL values in Table 5.

	ACDD01	ACDD02	ACDD10	ACDD11	ACDD12	ACDD20	ACDD21	ACDD22
AIC	130108	130322	129618	127419	127414	129312	***127405***	129113
BIC	130134	130357	129644	127454	127457	129347	***127449***	129166
LL	-65051	-65157	-64806	-63706	-63702	-64652	***-63698***	-64551

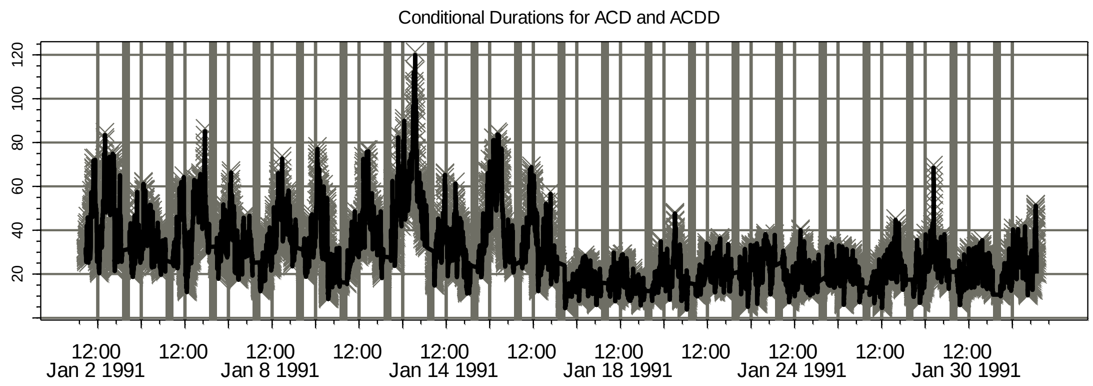
Table 5: AIC/BIC/LL-values for SEMIFAR(2)-ACDD Models

Table 6 summarises the parameters and diagnostics for variance equation of the SEMIFAR(2)-ACDD(2,1) model as selected. Though the JB-statistic still rejects normality, the LM-statistic is no longer significant indicating insignificant arch effects in the residuals.

	A0	ARCH(1)	ARCH(2)	GARCH(1)	LB-stat	LM-stat	JB-stat
SF(2)-ACDD(2,1)	0.0093	0.0796	-0.0256	0.9376	7.1446	18.5463	256.3364
p.value	0.0000	0.0000	0.0000	0.0000	0.8479	0.1001	0.0000

Table 6: SEMIFAR(2)-ACDD(2,1) parameters

In Figure 5, the black lines depict the conditional durations from the SEMIFAR-ACDD and SEMIFAR-ACDD models, whereas the grey crosses depict the conditional durations from the ACD and ACDD models in both panels. The plots are very similar but not identical, indicating that the alternative ACD models i.e. the ACDD and SEMIFAR-ACDD models are not only equivalent but are more adequate models of the signed duration process as supported by the lower residual statistics (as listed in Tables 4, 5 and 6).



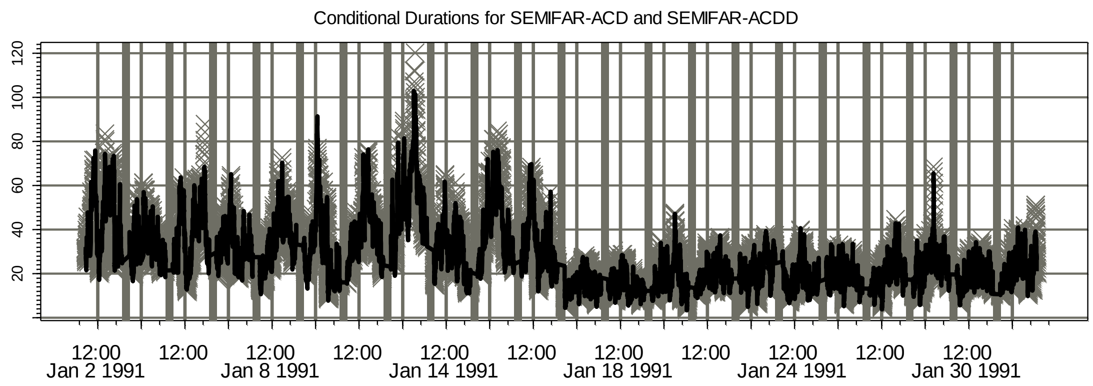


Figure 5: Conditional Durations Plots (ACD, ACDD, SF-ACD and SF-ACDD)

To make highlight the subtle differences in the conditional durations as captured by the various models, Figure 6 provides two scatter plot, one for the ACD/ACDD conditional durations and the other for the SEMIFAR-ACD/ACDD conditional durations. As can be seen in Figure 6, the SEMIFAR models fit very different conditional durations which must be the effects of dependencies in the mean series when the bid-ask dynamics have been embedded.

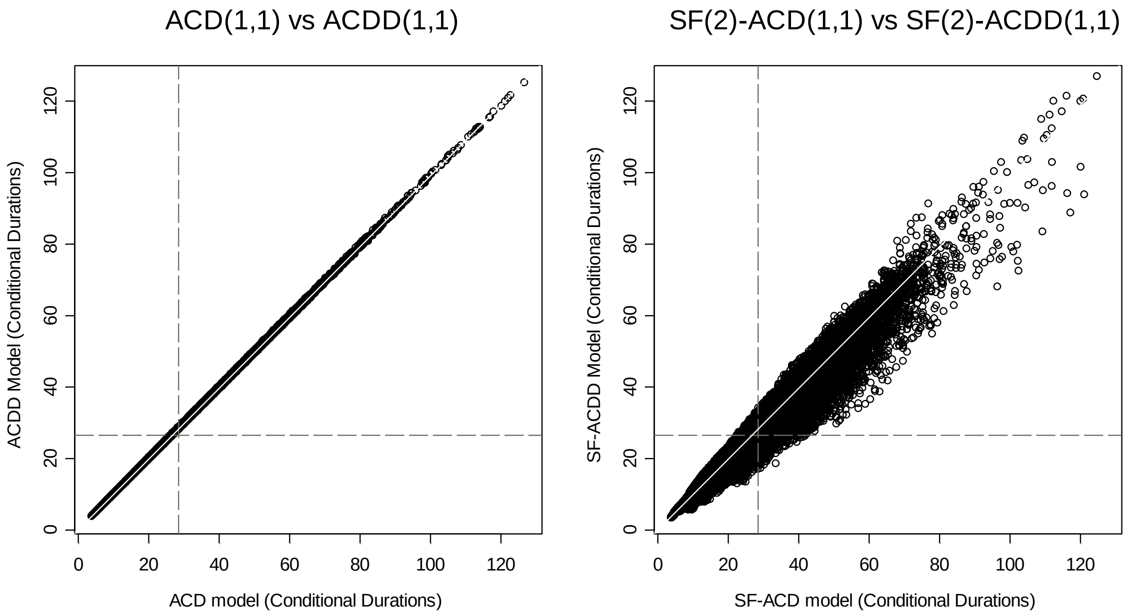


Figure 6. Scatter plots of the conditional durations.

Although there are not much differences between the conditional durations of the ACD and ACDD models, there are significant differences between the SEMIFAR versions of the same. The SF(2)-ACDD(1,1) model tends to give lower conditional durations more often that the corresponding ACD(1,1) models. One can see this tendency exhibited in panel 2 of Figure 5, where the black lines (depicting SF(2)-ACDD(1,1) conditional durations) are generally bounded by the grey crosses (depicting SF(0)-ACD(1,1) conditional durations).

7 Conclusions

This paper modifies the standard ACD model into a SEMIFAR–ACDD model so that non-stationarity and long memory in durations data can be addressed and captured using a more parsimonious parameterisation. Asymptotic results on SEMIFAR-GARCH models as reported by Feng, Beran and Yu [10] are extended to the SEMIFAR-ACDD model. The important property that the estimates of the SEMIFAR and ACDD parameter vectors are independent of each other, allows us to apply the data driven SEMIFAR algorithms to estimate the trend and the SEMIFAR parameters in the SEMIFAR–ACDD model. The ACDD parameters from the approximated ACDD innovations are obtained by inverting the SEMIFAR residuals.

If the fitted ACDD models are adequate, then the standardised residual innovations should behave as an IID sequence of random variables with the assumed distribution. In particular, if the fitted model is adequate, both the series $\{\varepsilon_i\}$ and $\{\varepsilon_i^2\}$ should have no serial correlations. The AIC/BIC/LL selected SEMIFAR(2)-ACDD(2,1) model resulted in residuals that had not only small but insignificant values of Ljung-Box and Lagrange-Multiplier statistics indicating strong model adequacy. The shape parameter of the GED distribution for the standardised residuals was 2.18 with a small Jarque-Bera statistic of 251.4 in addition to having mean and standard deviation estimates as assumed (i.e. 0.0 and 0.9937 against the model assumptions of $E(\varepsilon_i)=0$ and $E(\varepsilon_i^2)=1$).

The results indicate that the proposed SEMIFAR-ACDD representation can be used to capture both first-order and second-order dependencies in signed durations data. Further possible extensions to the ACDD model include leverage effects and the full range of GARCH-type extensions that are not readily available to the standard ACD model.

Acknowledgements: D.E. Allen thanks the Australian Research Council for research funding support and N. Jeyasreedharan thanks the Faculty of Business, University of Tasmania for funding to attend the Modelling and Managing Ultra-High Frequency Data: An International Conference, Perth, WA (MMUHFDIC, 2008). The authors are also grateful to Professor Mardi Dungey, University of Tasmania and the editor and anonymous referees for their invaluable comments and suggestions.

References

[1] L. Bauwens, and P Giot, The logarithmic ACD model: an application to the bid-ask quote process of three NYSE stocks, Annales d'Économie et de Statistique, 60, (2000) 117-149.

[2] L. Bauwens, and P Giot, Asymmetric ACD models: Introducing price information in ACD models, Empirical Economics, 28, (2003) 709-731.

[3] J. Beran, and Y Feng, Iterative plug -in algorithms for SEMIFAR models—definition, convergence and asymptotic properties, Journal of Computational Graphical Statistics, 11, (2002) 690-713.

[4] J. Beran, and Y Feng, SEMIFAR models—a semiparametric framework for modelling trends, long-range dependence and nonstationarity, Computational Statistics and Data Analysis, 40, (2002) 393-419.

[5] T. Bollerslev, Generalized autoregressive conditional heteroskedasticity, Journal of Econometrics, 31, (1986) 307-327.

[6] D. Creal, S.J. Koopman, and A Lucas. Generalized autoregressive score models, Journal of Applied Econometrics, 28 (5) (2013) 777-795.

[7] Easley D, and M. O'Hara. Time and the process of security price adjustment. Journal of Finance, 19, (1992) 69–90.

[8] R.F. Engle, Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation, Econometrica, 50, (1982) 987-1007.

[9] R.F. Engle, and J R Russell, Autoregressive conditional duration: A new model for irregularly spaced transaction data, Econometrica, 66, (1998) 1127- 1162.

[10] Y. Feng, J Beran, and K Yu, Modelling financial time series with SEMIFAR-GARCH model, IAM Journal of Management Mathematics, 18, (2007) 395-412.

[11] J. Grammig, and K O Maurer, Non-monotonic hazard functions and the autoregressive conditional duration model, Econometrics Journal, 3, (2000) 16-38.

[12] C.W.J. Granger, and R Joyeux, An introduction to long-memory time series models and fractional differencing, Journal of Time Series Analysis, 1, (1980) 15-30.

[13] N. Hautsch, Modelling irregularly spaced financial data: theory and practice of dynamic duration models (2004) Springer, London.

[14] J.R.M. Hosking, Fractional differencing, Biometrika, 68, (1981) 165-176.

[15] D.B. Nelson, Conditional heteroskedasticity in asset returns: a new approach, Econometrica, 59 (2) (1991) 347–370.

[16] M. Pacurar, Autoregressive conditional duration models in finance: a survey of the theoretical and empirical literature, Journal of Economic Surveys, 22 (4), (2008) 711 – 751.

[17] N. Shephard, Stochastic Volatility: Selected Readings. Oxford University Press: Oxford, (2005).

[18] M.Y. Zhang, J R Russell, and R S Tsay, A nonlinear autoregressive conditional duration model with applications to financial transaction data, Journal of Econometrics 104, (2001) 179-207.

[19] M.Y. Zhang, J R Russell, and R S Tsay, Determinants of bid and ask quotes and implications for the cost of trading, Empirical Finance, 15 (4), (2008) 656-678.